Stochastic Market Cycles and Option Valuation*

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ABSTRACT

A new Jump-Diffusion model of security price evolution is proposed. The model posits that asset price processes can be decomposed into a deterministic drift, a Wiener process, and two compound Poisson processes representing discontinuous price movements due to the arrivals of "good" and "bad" news during periods of economic expansion and contraction. Expansionary periods–Bull Markets–are characterized by more frequent arrival of "good news" that cause large price increases (jumps). Contractionary periods–Bear Markets–are characterized by more frequent arrival of "bad news" that cause large price decreases. Market conditions change at random points in time causing a change of regime. During each market epoch, the jump magnitudes are determined by random draws from stationary distributions. This form of information dynamics permits a simple form of returns' predictability, where conditional on the current phase of the market, the relative frequency and the direction of price movements may be anticipated. We derive the option pricing formula for this new price process and investigate the pricing bias due to misspecification error. We then generalize the model to allow for the rate of news arrival and the time of regime change to vary stochastically over time. We also outline several strategies for the estimation and simulation of the proposed model but cannot report any results at this time as this work is still in progress.

Twixt optimist and pessimist
The difference is droll:
The optimist sees the doughnut,
The pessimist, the hole.   McLandburgh Wilson, *Optimist and Pessimist*

Introduction

The financial markets recurrently experience 'mood swings,' brought about by the arrival of unexpected news that change investors' economic outlook, resulting in cycles of Bull and Bear markets. Indeed, the real world, the world of investors and the financial commentators, is replete with the optimistic Bull, who is disposed, despite grim circumstance, to expect flourishing markets, and the pessimistic Bear, who takes the demise of the market as evident, both of whom switch and exchange sentiments at every turn of the market. These fluctuations in investors' outlook may be the cause or the consequence of broader economic cycles, driven by recurrent technological change and innovation, or perhaps unexpected shifts in social, demographic, and political cycles, such as the congressional and presidential elections. Whatever their source, casual observation and empirical examination of returns evolution suggest that cycles with irregular length and depth are common features of many financial time series and a source of limited predictability in asset returns.

The prime examples of cycles in returns are the calendar based 'anomalies': The January effect (Banz 1981), Day-of-the-week-effect (Gibbons & Hess 1981), Time-of-the-moth effect (Hensel and Zeimba 93), Short-term positive serial correlation (Lo & Mackinlay 1988), Long-term negative serial correlation (Fama & French 1988), and others; all are suggestive of regular and irregular cycles.[1] Calendar based anomalies, particularly those with shorter duration, are not solely driven by sentiments, though the coincidence of investor pessimism and optimism could accentuate their magnitude (on this point see De Bondt & Thaler (1989)) and the references there in). Theoretical work in finance scarcely accords the pessimist and the optimist a mention; irrational emotions are difficult to incorporate into any theory of rational behavior. But Bulls and Bears do appear to exist in the financial markets. Hence this paper: an attempt to bring the two worlds closer. We reach this aim not by modeling what drives optimism and pessimism, but rather by expanding the existing models of return processes to permit the existence of periods of Bear and Bull markets. We then consider the consequence of the proposed return process for the theory of option valuation.

To date, the empirical evidence in finance documents several important characteristics of stock

returns. First, relative to the normal density, the empirical distribution of returns is leptokurtic or fat tailed (see Tucker (1992) and finance journal papers in this area). Various factors can explain the existence of fat tails. Prominent among them are random changes in the volatility of returns and discrete price movements resembling jumps, Stochastic volatility models and jump-diffusion process have been proposed so as to allow for leptokurtosis.

In the option pricing literature, building on the compound poison model of Press (1967), Merton 1976b extended the Black and Schole's model 1973 so that price movements include a jump component. Merton 1976a showed that significant option pricing error can occur if the jump component of returns is overlooked. The existence and importance of the jump component for a variety of asset prices (returns) have subsequently been confirmed by many researchers.[2].

Models for pricing options under stochastic volatility were proposed by Wiggins (1987), Johnson & Shanno (1987), Scott (1987), and Wiggins (1987). Hybrid models allowing for both stochastic volatility and jump-diffusion have recently been proposed by Bates (1995b), Scott (1995), and Ho, Perraudin & Sorensen (1996). New models suggesting alternative returns distribution have also appeared in the recent literature. These include the implied binomial trees of Rubinstein (1994), stable paretian model of McCulloch (1987), the variance-gamma of Madan & Seneta (1990) and the gamma processes of Heston (1993), and finally the jump in volatility model of Naik (1993). Bates (1995d) surveys the extant empirical literature on the validity of alternative models. He concludes that tests of stochastic volatility option pricing models from time series data "are still in an early stage, and far from conclusive." On the hand there is stronger evidence in support of the existence of a jump component, which seems consistent with the large "event studies" literature and most recently corroborated by the evidence presented in Ederington & Lee (1993) regarding information flows associated with firm-specific and macroeconomics factors.

There is also an overwhelming volume of evidence that points to various forms of predictability in asset returns.[3] The accumulated research generally documents the calendar based anomloies noted above. Predictability has been attributed to a variety of factors, including variations in risk premium over time due to changes in economic conditions brought about by business cycles, and market inefficiencies that lead to persistence of arbitrage opportunities. While time-varying expected returns can in principle be consistent with an efficient market, the persistence of arbitrage opportunities may suggest some degree of irrationality on the part of investors. Despite disagreements as to the source of predictability, there is strong consensus that predictability is a genuine feature of financial returns, though as Pesaran & Timmermann (1995) indicate, attempts to interpret excess return predictability are model-dependent and hence inconclusive.

The literature on integrating predictability into asset price processes and option pricing models is new and relatively sparse. Lo & Wang (1995) consider the consequences of predictability in underlying asset's return for pricing options. They proposed a continuous time model of asset

price processes which permits *local* predictability generated by a stochastic component in the drift of a mean-reverting model for the log-price process. In a manner similar to Merton 1976*a*, they showed that ignoring this form of predictability results in significant option pricing error. Lo & Wang (1995) provided a simple adjustment to Black & Scholes (1973) formula to account for predictability. However, their model lacked any jump component suggesting that predictability was solely due to correlation in instantaneous movements in prices.[4]

The purpose of this paper is to extend the preceding literature by proposing a jump-diffusion model of returns which explicitly permits Bull and Bear markets. The proposed process permits a simple form of predictability, which is consistent with the efficient market hypothesis. In particular, we model predictability by the ability to assign, *ex ante*, relative probability to up and down price jumps during specific market periods. That is, conditional on knowing whether the economy is in an expansionary or a contractionary phase, one expects a higher frequency of good or bad new. Of course, the market phase changes randomly. The most innovative feature of the proposed model is the fact that it explicitly accounts for the existence of unexpected changes in investors' sentiments that lead to boom and bust periods, with large price movements that may not accord with changes in the 'market fundamentals'.

The proposed jump-diffusion model accommodates all important features of returns that have been documented empirically. The addition of a jump component permits leptokurtosis and random shifts in volatility of returns. The information dynamics of the jump components can admit a rich menu of *ex post* autocorrelation in returns. The proposed process, however, can also generate a high degree of unpredictable behavior; even if the parameters of jump-diffusion process are known, the timing of shifts in market conditions are stochastic. Given this general specification for an asset's price process, we derive the pricing formula for an European type option using both general equilibrium and arbitrage arguments. investigate the nature of pricing error due to misspecification of the process. We then generalize the model in two important directions: First, we permit the rate of information arrivals to change stochastically over time. Second, we consider candidate stochastic processes that could govern the switch time from one market regime to another.[5]

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The Model

To highlight the impact of market cycles on option pricing, when deriving the pricing formula for the proposed jump-diffusion process, we retain a number of assumptions that are standard to option pricing literature. Briefly, we assume "frictionless continuous markets," where there are no taxes, transactions costs, or restrictions on borrowing and short selling. The short-term interest rate, denoted by r, is assumed to be constant over time, and identical whether investors borrow or lend funds. It is assumed that the underlying stock does not pay dividends during the life of the option[6] and that the continuous part of the price process follows a geometric Brownian motion.

Merton 1976b extended the Black and Scholes model by assuming that returns are generated by a mixture of continuous and jump processes. The jump component measures abnormal change in prices due to arrival of news that may be specific to the firm or an industry. The discrete points in time when new information arrives are assumed to be random and driven by a Poisson process. Given that important information arrives, the magnitude of the price jump is determined by drawing from a distribution. Jump magnitudes from successive drawing are assumed to be independent and identically distributed.

We extend Merton's 1976b model in a number of important directions. First, a distinction is made between 'good' and 'bad' news, defined as any information that leads to non-marginal price increases (up-jumps) or decreases (down-jumps), respectively.[7] We define a boom period–Bull market–as an interval of time characterized by higher frequency of good news arrivals. Although adverse information can arrive at any point in time, such events are assumed to be less likely during boom periods. This type of information dynamic may be a consequence of an expanding economy.

The bust period–Bear market–is defined as an interval of time characterized by higher frequency of bad news arrivals. Again favorable information may arrive at any point in time, though such events are assumed to be less likely during the recessionary or bust periods. This type of information dynamic may be a consequence of a contracting economy. In short, we allow the mean number of arrivals for good and bad news to be different during the Bull and Bear markets.

Second, we allow market conditions to shift at random points in time causing an unanticipated change of regime. Switches in market conditions may be related to social and political events, 'herd' behavior resulting from analysts forecasts, unexpected arrival of significant adverse or favorable news during a boom or bust period, which could lead to reversal in the direction of price movements, technological innovations, natural disasters, or factors associated with business cycles.

The proposed jump-diffusion structures provide a simple interpretation of stock price dynamics: The total change in price is due to a deterministic growth rate per unit of time plus two stochastic components. The first source of randomness is 'local' price variations that are due to normal

market conditions. The second source of randomness is due to the arrival of news which leads to 'abnormal' movement in prices. We posit that the arrival of good (bad) news lead to price increases (decreases) and that there are periods when good news (bad news) arrive more frequently. Moreover, we assume jump magnitudes during and across each market epoch are independent.

To formalize these notions, let the price process, $S(t)$, contain two stochastic components; a geometric Brownian motion with constant variance, σ^2, and two "Poisson-driven" components in continuous time.[8] Let the counting processes $\{N_p^e(t), t \geq 0\}$ be independent Poisson processes with intensity parameters λ_p^e, where for the balance of the paper index $e = G, B$ stand for good and bad news, and index $p = b, c$ signify boom and contractionary periods respectively.

Let $[0, T]$ be any arbitrary interval of time containing exactly one boom and one contractionary periods. Let $t_s \epsilon [0, T]$ be the time when the switch in the regime occurs. Suppose in the time interval $[0, t_s]$ the arrival rate for good news exceed the arrival rate for bad news. Such a period will be called a *Bull Market* and formally defined as $\lambda_b^G > \lambda_b^B$ and $t \epsilon [0, t_s]$. This formulation assigns a higher probability to good news arriving during the boom periods and therefore provides a formal description of market optimism.

Similarly the *Bear Market* period is defined as one in which $\lambda_c^G < \lambda_c^B$ and $t \epsilon (t_s, T]$. Again this formulation provides a formal description of market pessimism. Since the arrival rates determine the probability of various types of events over 'small' time intervals, these definitions simply assign higher probability to up and down jumps during each market epoch. Our formulation also assures that both types of news can arrive during the Bull and Bear markets.[9] It is important to note here that a Bull-Bear market sequence is the basic building block for longer time horizons, which will contain many expansions and contractions. Hence, we focus first on a period with one sequence of boom and bust periods. We then will generalize our results to many such periods. It will prove useful to define two indicator function $I_b = I_{(t \leq t_s)}$ and $I_c = I_{(t > t_s)}$, which assume the value 1 during the Bull and Bear markets respectively and zero otherwise. These indicator functions may be indexed by j, which would distinguish the j-th boom or bust periods.

The parameters λ_p^G and λ_p^B, $p = b, c$, measure the mean arrival rates of good and bad news *per unit of time* during the boom and contractionary periods. The total number of news arrivals of each type up to time t, denoted by $N^e(t)$, $e = G, B$, has a Poisson distribution with mean $\lambda^e(t)$, and for $t, t + s \epsilon [0, T]$,

$$\text{Prob.}\{N^e(t + s) - N^e(s) = n\} = e^{-\lambda^e(t)} \frac{(\lambda^e(t))^n}{n!},$$

where $\lambda^e(t) = \lambda_b^e(t \wedge t_s) + I_c \lambda_c^e(t - t_s)$ and $E[N^e(t)] = \lambda^e(t)$ for all e. The time between news arrivals are assumed to be independent and identically distributed random variable for all processes. Moreover, it is assumed that the two Poisson processes are independent. This implies that the probability of multiple events occurring at one point in time, i.e. good and bad news arriving

simultaneously, is zero.[10]

Upon the arrival of each type of information–the occurrence of the Poisson events–the magnitude of price change, y_e, $e = B, G$, is determined by drawing from two stationary distributions ; $y_G \sim (\mu_G, \delta_G^2)$ and $y_G \geq 1$, and $y_B \sim (\mu_B, \delta_B^2)$ and $y_B \epsilon (0, 1)$. Note that while up jumps can assume any positive values, the down jumps are restricted to a maximum of -100% which is consistent with stocks' limited liability. Jump magnitudes within and across market epochs are assumed to be independent and identically distributed. For simplicity it is assumed that the distribution of jump magnitudes for both G and B are the same during the bull and the bear markets.[11] Finally, the *percentage* change in price upon the arrival of new information is denoted by $k_e = \mathrm{E} y_e - 1$, and its mean value by $\bar{k}_e = \mathrm{E} k_e$, where E is the expectation operator with respect to the distribution of y_e: $k_G > 0$ and $-1 < k_B < 0$. Note that so far we have only restricted the supports of y_e. Explicit distributional assumptions will only be needed when we derive a tractable option pricing formula.

The stock price process posited above will be a mixed stochastic differential-difference equation:

$$\frac{dS}{S} = (\mathcal{A}_t)dt + \sigma dZ + k_G dq_G(\lambda_p^G; t_s) + k_B dq_B(\lambda_p^B; t_s) \tag{1}$$

where S is the stock price, $\mathcal{A}_t$ is the jump-adjusted drift of the process; σ is the instantaneous variance when the Poisson events do not occur; dZ is a Wiener process; dq_es are the Poisson processes that derive the dynamics of information arrival during the boom and contraction periods; $dq_e = 1$ when good (bad) news is unexpectedly revealed and zero otherwise, and each process assumes a different arrival rates during the boom and bust periods. The time subscript on $\mathcal{A}$ is used to emphasize the fact that the optimistic and pessimistic outlook associated with boom and bust periods are captured in the drift of the process. That is let α denote the drift of the process if there were no jump components. Then $\alpha_b = \alpha - \lambda_b^G \bar{k}_G - \lambda_b^B \bar{k}_B$ is the jump adjusted drift during the Bull market ($t \leq t_s$) and $\alpha_c = \alpha - \lambda_c^G \bar{k}_G - \lambda_c^B \bar{k}_B$ is the jump adjusted drift during the subsequent Bear market ($t_s < t \leq T$), and $\mathcal{A}_t = \sum_p \alpha_p I_p$, where $p = b, c$. Hence, relative to the standard geometric Brownian motion, the drift of the process in (1) will change change with the phases of the market, and can on any value depending the arrival rate of the good and bad news and the mean jump magnitudes. This is an important innovation relative to other existing models which assume a constant drift . In particular, it will be shown below that while the drift of the continuous part of the process will not effect option prices, as is expected, the jump component of the drift will have an important and fluctuating influence on option prices. Getting ahead of ourselves a bit here, we note that, as is the case with "predictability" (Lo & Wang 1995), cyclical market movements also effect the drift of the underlying price process. However, the impact on option prices is significantly more pronounced.[12]

We obtain a solution for our proposed stochastic difference-differential equation:[13]

$$S(t) = S \exp[(\alpha - 0.5\sigma^2)t - \Phi + \sigma Z(t)] \, Y^G(N^G) \, Y^B(N^B) \qquad (2)$$

where

$$\Phi = (\lambda_b^G \bar{k}_G + \lambda_b^B \bar{k}_B) \, (t \wedge t_s) + I_c(\lambda_c^G \bar{k}_G + \lambda_c^B \bar{k}_B) \, (t - t_s)$$

and for $p = b, c$, and $e = G, B$;

$$Y^e(N^e) = \begin{cases} 1 & \text{if } N^e = 0 \\ \prod_{i=1}^{N^e} y_i^e & \text{if } N^e = 1, 2, 3, \ldots \end{cases}$$

where the N_p^e are the counting Poisson processes described above and subscript i is the i-th jump of type e during the market epoch p.

Using equation (2), the s period rate of return starting at $t = 0$, $r(s)$, is defined as:

$$r(s) = \ln[\frac{S(s)}{S}] = [(\alpha - 0.5\sigma^2)t - \Phi + \sigma Z(t)] + \sum_i^{N^G} \ln(y_i^G) + \sum_i^{N^B} \ln(y_i^B)$$

In the present form, the distribution of returns are unspecified and critically depends upon the choice of distribution for the jump magnitudes. Following Merton (1976b) it is standard to assume y_i's have independent log-normal distribution. This assumption, however, is inappropriate in our model given the fixed supports of the distribution for good and bad news. There are a number of candidate distributions with the appropriate minimum and maximum. In particular the Beta distribution (domain lies on 0 to 1) is ideal for bad news jumps and, the Pareto distribution (domain 1 to infinity) would work well for the good news jumps. Other candidates included the uniform , truncated log-normals or a mixture of such distributions. The choice of the distribution for the jump magnitude has important implication for the kurtosis and skewness of the return process; by choosing a mix of distributions one can generate any form of leptokutosis. In the option pricing context, as described below, no distributional assumptions are required unless one wishes to simplify the derived option pricing formula. This issue will be revisited in later sections of the paper. It is useful to note that process has independent and stationary increments and its *conditional* moments are:

$$E[r(t)] = [(\alpha - 0.5\sigma^2)t - \Phi + \sum_e \lambda^e E(\ln(y^e)),$$

$$Var[r(t)] = [(\sigma^2)t + \sum_e \lambda^e Var(\ln(y^e)),$$

$$Cov[r(t_2) - r(t_1), r(t_4) - r(t_3)] = 0, \quad t_1 < t_2 \le t_3 < t_4.$$

Option Valuation

Given the dynamics of the stock price described above, an option formula can be derived under two alternative hypothesis. First, if the process describes the evolution of returns for an individual asset, and if the the risk associated with the jump components are uncorrelated with returns on the market protfolio (nonsystematic), then jump risk is not priced in equilibrium and one can follow Merton (1976b) and equate the option value to the expected value of its payoff discounted at the riskless rate. This parallels the replication arguments of Black & Scholes (1973), and followed by citeasnounCoxRos76 and Jones (1984), who assume fixed jump magnitudes. Of course, when jump magnitude is stochastic, the Balck-Scholes startegy only holds in an expectational sense and is neither replicating nor self financing. Indeed, Naik & Lee (1990) have shown that when there is jump risk, the costs of maintaining the Balck-Scholes hedging startegy can be prohibitive.

Alternatively, if the risk associated with the jump components are systematic and non-diversifiable, as is the case for a portfolio of assets or the market portfolio, then general equilibrium arguments must be used to price options. The advantage of the equilibrium approach is that one can derive the Arrow-Debreu price of 'jump insurance', and construct the asset price process under the risk neutural probability measure. This is fundamental to option valuation, since theoretical option prices are the discounted expted value of their payoffs under the risk neutural measure. The equilibrium approach, however, requires a number of strong assumptions about the structure of the economy (representative agent), the technology (linear), and preferences (typically time-separable power utility). To obtain a tractable option pricing formula, additional assumptions regarding the distribution of jump magnitudes must be invoked under both approaches.

In this paper we develop an option valuation formula for the proposed market cycle process under either approaches. First, we develop our formula using the Black-Scholes replication principle and study the implications of pursuing a continuous hedging strategy under our proposed process. Next we derive our general equilibrium valuation formula for an economy identical to that assumed in Bates (1991) and study how the equilibrium price of risk varies with the Bull and Bear market cycles.

Continuous Hedging and Market Cycles

Given the proposed asset price process in (1), we can derive the dynamics of the option price using Ito's lemma and its counter part for jump processes (see Merton (1990, Chapter 3)). Let the option price $F(S, t)$ be a twice-continuously differentiable function. If S follows the dynamics posited in equation (1), then the option's return dynamics can be written as:

$$\frac{dF}{F} = (\mathcal{A}_t^F)dt + \sigma_F dZ + k_G^F dq_G^F(\lambda_p^G; t_s) + k_B^F dq_B^W(\lambda_p^B; t_s) \tag{3}$$

where the instantaneous expected return on the option, $\mathcal{A}_t^F = \sum_p \alpha_p^F I_p$, varies with the phase of the market: $\alpha_b^F = \alpha^F - \lambda_b^G \bar{k}_G^F - \lambda_b^B \bar{k}_B^F$ during the Bull market ($t \leq t_s$) and $\alpha_c^F = \alpha^F - \lambda_c^G \bar{k}_G^F - \lambda_c^B \bar{k}_B^F$ during the Bear market ($t_s < t \leq T$); and α_F is the mean return on the option and σ_F is variance of return on the option if there were no jump components, $\bar{k}_F^e = \mathrm{E}y_F^e - 1$ is the expectation of the random percentage change in the option price when information about the underlying stock arrives, and the remaining symbols are defined as before. Ito's lemma and its analog for jump processes suggest the following relationships:

$$\alpha_F = \frac{\{0.5\sigma^2 S^2 F_{SS} + (\mathcal{A}_t)SF_S + F_t + \omega^G \mathrm{E_G}(\Delta F^G) + \omega^B \mathrm{E_B}(\Delta F^B)\}}{F}$$

$$\sigma_F = \frac{\sigma SF_S}{F}$$

where $\Delta F^e = (F(Sy^e, t) - F(S, t))$, $\omega^e = \sum_p \lambda_p^e I_p$ for $e = G, B$, and subscripts on $F(\cdot, \cdot)$ denote partial derivatives. Note that the dynamics of option price jumps are perfectly, but *not linearly*, dependent upon the stock price jumps. That is as stock price moves with the phases of the market, so does the option price. In particular, changes in the drift of the stock price ($\mathcal{A}_t$) and the differential rates of news arrival during the Bull and the Bear markets, effect the drift of the option price (eq. 3A) as we move from one phase of the market to another.

To drive an option pricing formula we consider a self financing portfolio strategy with portfolio weights w_i, $i = 1, 2, 3$, placed in the stock, the risk free asset, and the option, where $\sum_i w_i = 1$. Let π be the values of this portfolio. Then the return dynamics of π can be written as:

$$\frac{d\pi}{\pi} = (\mathcal{A}_t^\pi)dt + \sigma_\pi dZ + k_G^\pi dq_G^\pi(\lambda_p^G; t_s) + k_B^\pi dq_B^\pi(\lambda_p^B; t_s) \tag{4}$$

where $\mathcal{A}_t^\pi = \sum_p \alpha_p^\pi I_p$, $p = b, c$, is the market phase dependent expected return on the portfolio, and $\alpha_p^\pi = \alpha^\pi - \lambda_p^G \bar{k}_G^\pi - \lambda_p^B \bar{k}_B^\pi$); and α_π is the mean return on the portfolio and σ_π is variance of return on the portfolio if there were no jump components, $\bar{k}_\pi^e = \mathrm{E}y_\pi^e - 1$ is the expectation of the random percentage change in the portfolio price when information about the underlying stock arrives. Again, note that the drift of the portfolio changes over time. From the dynamics of the stock price (1), the option price (3), and the dynamics of the risk free asset we can deduce that $\alpha_\pi = w_1(\alpha - r) + w_2(\alpha_F - r) + r$, $\sigma_\pi = w_1\sigma + w_2\sigma_F$, and $y_\pi^e = w_1(y^e - 1) + w_2(\Delta F^e)F^{-1}$.

As is the case in Merton's1976b, the introduction of jumps implies that the portfolio can not be made riskless, as a set of portfolio weights $\{w_1^e, w_2^e\}$ that would eliminate the jump risks, forcing $y_\pi^e = 1$ for $e = G, B$, does not exist (there are more sources of risk than can be hedged with two assets). Hence since a perfect hedge portfolio can not be formed, no-arbitrage arguments cannot be used to price options. Instead we adopt Merton's 1976b strategy and assume that the jump components of the stock return are uncorrelated with the return on the market portfolio. This implies that the β of the self financing portfolio will be zero and under the assumptions of CAPM,

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the expected return on the portfolio must be the risk free rate. Under these assumption, the portfolio weights w_1, w_2 will be identical to those in the Black and Schole's model and the option price must satisfy the following partial differential-difference equation:

$$0.5\sigma^2 S^2 F_{SS}(S, \tau) + (r - \theta)S F_S - F_\tau - rF + \lambda^G \mathrm{E_G}(\Delta F^G) + \lambda^B \mathrm{E_B}(\Delta F^B) = 0 \tag{5}$$

where $\theta = (\sum_p \lambda_p^G I_p)\bar{k}_G + (\sum_p \lambda_p^B I_p)\bar{k}_B$, $\tau = T - t$ is the time to maturity of the option and $F(0, \tau) = 0$ and $F(S, 0) = \max(S_T - K)$ are the boundary conditions for a call option. Note that the risk free rate r is used as the 'equilibrium' instantaneous expected rate of return on the portfolio, as would be the case in a risk neutral economy. Moreover, it should be clear that the portfolio replicates the payoffs to the option in an expectational sense, where infussion (withdrawal) of funds occur when the stock price jumps. Indeed, as Naik & Lee (1990) have shown, in the case of single jump model of Merton (1976b), the funds needed to replicate the payoff to the option can be substantial, increasing with the volatility of jump magnitudes, and decreasing if the hedge portfolio is based on the conditional variance of the jump-diffusion process ($\sigma^2 + \lambda\sigma_y^2$ in their model) rather than the variance of the Brownian motion component (σ). Our model suggests that relative to the case studied in Naik & Lee (1990), additional (less) infussion of funds would be needed during the Bull (Bear) market to replicate the payoffs to the call option.

To obtain a solution to (5) we follow the steps in Merton (1976b, p319) and obtain the pricing formula:

$$F(S, \tau) = \sum_{n^G=0}^{\infty} \sum_{n^B=0}^{\infty} \frac{e^{-\lambda^G}(\lambda^G)^{n^G}}{n^G!} \frac{e^{-\lambda^B}(\lambda^B)^{n^B}}{n^B!} \times \tag{6}$$
$$\mathrm{E}_{n^G, n^B}\{W(SX(n^G)X(n^B)e^{-\Omega}, \tau; K, \sigma, r)\}$$

where

$$\lambda^e = \lambda_c^e(\tau \vee t_s) + I_b\lambda_b^e(t_s - t)$$

$$\Omega = (\lambda_c^G \bar{k}_G + \lambda_c^B \bar{k}_B)(\tau \vee t_s) + I_b(\lambda_b^G \bar{k}_G + \lambda_b^B \bar{k}_B)(t_s - t)$$

$$X(n^e) = \begin{cases} 1 & \text{if } n^e = 0 \\ \prod_{i=1}^{n^e} y^e & \text{if } n^e = 1, 2, 3 \ldots \end{cases}$$

where n^e–the number of news of type $e = G, B$–is Poisson distributed with parameter λ^e, and $W(\bullet; \bullet)$ is Black-Schole's formula for stock price S, time to maturity τ, exercise price K, volatility σ and interest rate r.

So far the only assumption made with regards to the jump magnitudes for both the good and bad news was to fix the support of their distributions. Without additional distributional assumption, one cannot evaluate the expectation with respect to jump magnitudes ($\mathrm{E}_{n^G+n^B}\{\bullet\}$) and therefore formula (6) may not be simplified further.[14]

Equilibrium Pricing of Jump Risk

The risk neutral distribution implicit in the actual prices of the assets is the most important ingredient for calculating theoretical prices for both European and American options written on that asset. In essence the risk neutral distribution summarizes the prices of the arrow-Debreu state-contingent claims. It is fairly straight forward to obtain the risk neutral distribution for standard continuous processes via arbitrage arguments (Grundy 1991). As it is well known arbitrage arguments cannot be used to achieve the same aim when the underlying price moves according to a jump-diffusion or stochastic volatility process. For these and more complicated process, as proposed in this paper, deriving the risk neutral measure requires pricing the jump risk in a general equilibrium setting. This in turn requires additional assumptions about the economy, the technology, and agent's preferences, as well as some distributional assumptions.

Ahn & Thompson (1988) introduced discontinuous jumps into the very general and flexible model of Cox, Ingersoll & Ross (1985) and derived important results regarding the determination of asset prices. However, rather than assuming specific preferences and deriving explicit option pricing formulas, these authors focused on general pricing relationships and essentially reconsidered the results in Cox et al. (1985) for an economy with jumps. With a view to price options with discrete price movements, Naik & Lee (1990) assumed a representative agent with power utility function in a single good Lucas type economy and derived an option pricing formula that includes market price of jump risk, which itself depends upon the representative agent's preference for risk. Finally, Bates (1991) explicitly considered pricing of jump risk in a restricted version of Cox et al. (1985) model. In particular, Bates (1991) assumed that the jump component of aggregate wealth is distributed log-normally and that agents have state independent time-additive power utility functions. He then derived an option pricing formula very similar to that of Merton's 1976*b* but where the coefficients are explicit functions of the agent's risk preferences.

For this paper we have adopted all the assumptions in Bates (1991) with one exception; rather than assuming log-normal distribution for the jump component of the wealth process, we must assume distributions that would allow differential arrival rate for good and bad news during boom-bust periods, which is fundamental to our stochastic cycles explanation of the world. Our model provides interesting insights on how the price of jump risk varies with preferences over boom and bust periods, though these results critically depend upon our distributional assumptions. Unfortunately, at the time of this writing this work is incomplete and subject to some revision. We expect to complete this part of our analysis shortly. We note that it is a straight forward exercise to extend Bates (1991) results to a simple version of the market cycle model in which there is only a single jump component with log-normal distribution of jump magnitudes whose parameters change over time and hence generate boom to bust period. Although this model is

capable of generating market cycles, it violate our original intention to separate good and bad news, and our aim to explicitly model the asymmetry in the arrival rate and jump magnitudes of favorable and unfavorable news. We believe this asymmetry is critical to option valuation and our preliminary results support this conjecture. We provide evidence to support this conjecture in the final draft of our paper prior to the AFA meetings.

Stochastic Switch Times

In developing the option pricing model in (6), we assumed the regime switch time was fixed and known in advance. This is a strong assumption though there are circumstance when the time of regime change may be known, for example if the underlying price process is for a specific firm, the timing of changes in management or the introduction of new products may be known. However, in most circumstance investor do not known when exactly a turn in the market will occur. In such situation, the change in regimes can be modeled as regulated by a random point process, which will assign a probability to a turn in market conditions at any point in time.

Then, investors can only estimate the switch times, and the expected length of the boom and bust periods based on the available information. In such settings, the expected switch time $\hat{t}_s$ can be substituted in the option pricing formula in equation (6). Furthermore, if the available information is large enough so that investors can construct a reasonable distribution for switch times, i.e., t_s is a random variable with a known distribution $G(t_s)$, then the option price formula will become the expected value of $F(S, \tau; t_s)$ with respect to $G(t_s)$, where $G(t_s)$ may be discrete or continuous. These statements will be formalized in the next version of the manuscript.

Estimation and Specification Error

Merton (1976a) and Lo & Wang (1995) have discussed the problem of errors in option pricing due to the misspecification of the stochastic process generating the underlying stock's returns. Merton (1976a) addressed the following problem: suppose an investor believes that the stock price dynamics follows a continuous sample path geometric Brownian motion, and therefore he/she uses the Black-Scholes formula to calculate the option price when the true process for the stock price is described by a jump-diffusion process. How will the calculated values based on the misspecified process for the stock compare with the values based on the correct process? The basic results from Merton's simulations is that the incorrect model gives too low of a value for deep in- or out-of-the-money options, and it gives too high of a value for options which are nearly at-the-money. Hence, the deep out-of-the-money options based on the Black-Schole's formula which appear greatly "over-valued," may not be over-valued at all if the underlying stock process includes jumps.

Similar questions can be raised using the model proposed above. In particular, suppose the

investor uses Black-Scholes' or Merton's jump-diffusion formula to appraise the call option when the true process is the boom-contraction jump-diffusion process proposed above. In both cases, one is interested in knowing how the pricing error will behave.

We are currently conducting simulation studies to answer these questions. To provide a satisfactory answer, however, we plan to estimate the parameters of our model rather than assuming specific values chosen randomly. Bates (1991) provides estimates for the S&P-500 futures options for a single jump model prior to the crash of 1987. Bates (1995c) provides similar estimates for th post crash period. We plan to use the same data and the estimates reported by Bates to assess the validity of our model.

Briefly, the design of our experiment is as follows: Given the data on S&P-500 index for the Bull and Bear market periods adjacent to the crash of 87, we use the cumulant method discussed in Press (1967) to obtain starting values for our model. These parameter values will then be used as starting values to estimate our highly nonlinear option pricing formula using maximum likelihood estimation (MLE), as in Bates (1991). This step will provide us with coefficient estimates that are comparable with those reported in Bates.

Given the MLE parameter values for our model for the pre- and post-crash periods, the value of the index S_t and the *appropriate* interest rate r_t at point t prior to crash, and a range of exercise prices symmetrical to S_t, calculate option prices for the Black-Scholes, the single jump models of Bates, and the model proposed in this paper. This will enable us to directly compare option prices under each model with actual market prices. For the at-the-money options, use option price from the market cycle model as the bench mark (as though these were the 'true' market price of the option). For a given date, use actual interest rate (e.g., 8%), the exercise price for an at-the-money option, and the time to maturity (say 90 days) and solve for the implicit volatility using the Black-Scholes' model. Use this implicit volatility and the assumed interest rate and time to maturity, and a range of exercise prices (say 200 up to 400), to calculate out-of-the-money option prices for each strike price again using the Black-Scholes formula (this is what is done in practice since it is known that the Black-Scholes formula provides a good approximation for at-the-money options). Finally, compare the out-of-the-money option prices calculated in the previous step with those from the market cycle model and the single jump model. The rationale for this analysis is to assess the performance of each model when pricing out-of-the-money options and to see which model best captures the "skewness premium."

We plan other experiments in addition to those described above. In particular, we intend to use statistical procedures for identifying 'regime' changes to estimate the time of change in market phase from the data and to compare the estimated switch times with the actual data. In essence, these methods work iteratively to identify the switch time(s) that maximizes the likelihood function.

Extending the Option Valuation Model

General Results

The boom-contraction jump-diffusion model proposed above represents a simple structure for demonstrating the idea of market cycles and how they effect option valuation. The proposed model can be further generalized by assuming that the rate of information arrival and the mean jump for both bad and good news are functions of time. Invoking the same assumptions as in previous section and following similar steps we can derive a more general result, which we summarize in the following theorem.

Theorem: Let $\lambda_e(t), e = G, B$ be the intensity functions for the Poisson processes driving the up and down jumps that have fixed mean magnitude $\bar{k}_e$. Let $\lambda_e(t)$ be independent and identically distributed random variables. Then the stock return is governed by the following jump-diffusion process:

$$\frac{dS_t}{S_t} = (\mu(t) - \sum_e \lambda_e(t)\bar{k}_e)dt + \sigma dZ + \sum_e (k_e)dq_e(\lambda_e(t))$$

where all notation is defined as above. Alternatively, the process may be represented in its integral form:

$$S_t = S_0 \, \exp[\int_0^t \mu(s)ds - 0.5\sigma^2 t + \sigma Z(t)] \, \Pi_e Y_e(N_e) \tag{7}$$

Letting $S = S_t$ and following the continuous hedge arguments as above, then the option value, $F(S, \tau)$, must satisfy the following partial differential-difference equation:

$$0.5\sigma^2 S^2 F_{SS} + (r - \sum_e \lambda_e(t)k_e)SF_S - F_t + rF$$

$$+ \sum_e \lambda_e(t) \, \mathrm{E}_{Y_e(t)}(F(SY_e(t), t) - F(S, t)) = 0 \tag{8}$$

Using the boundary conditions as above, a solution for equation (8) is given by the following expression:

$$F(S, \tau) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} P_n(\Lambda_G(t))P_m(\Lambda_B(t)) \times$$

$$\mathrm{E}_{n,m} \, \{W[SX_G(n)X_B(m)e^{-\Theta_G(t) - \Theta_B(t)}, \tau; K, \sigma, r]\} \tag{9}$$

where

$$\Lambda_e(t) = \int_t^T \lambda_e(s)ds,$$

$$P_i(\Lambda_e(t)) = \frac{e^{-\Lambda_e(t)}(\Lambda_e(t))^i}{i!}, i = n, m$$

$$\Theta_e(t) = \bar{k}_e \int_t^T \lambda_e(s)ds = \bar{k}_e \Lambda_e(t),$$

and for $i = n, m$ and $e = G, B$,

$$X_e(i) = \begin{cases} 1 & \text{if } i = 0 \\ Y_e^i & \text{if } i = 1, 2, 3, \ldots \end{cases}$$

where the number of news arrivals $i = n, m$ are independent Poisson distributed with parameters $\Lambda_i(t)$.

Proof: The proof for this general case is given in the appendix. The proof for Bull-Bear market model presented in section 2, which assumed time invariant frequency of news arrival $(\lambda_e(t) = \lambda_e)$, and other parameterization described below are special cases of the proof given in the appendix.

The theorem gives the option value of a process with two independent Poisson driven jump processes. This can be easily generalized to a process with more Poisson driven jump processes. A particularly interesting case occurs when we permits the frequency of news arrival in Merton (1976b) to be a function of time. Then we have the following corollary.

Corollary: Adopt the same assumptions and notations as in the Theorem. Suppose the stock process follows a one-jump-diffusion process:

$$\frac{dS_t}{S_t} = (\mu - k\lambda(t))dt + \sigma dZ + (k)dq(\lambda(t))$$

Then the option value at time t is ,

$$F(S, T - t) = \sum_{n=0}^{\infty} P_n(\Lambda(t))EW[SX(n)e^{-\Theta(t)}, \tau; K, \sigma, r]$$

where $\Lambda(t) = \int_t^T \lambda(s)ds$ and $\Theta(t) = \bar{k}\Lambda(t)$.

Further Generalizations

There is a rich variety of special cases of the general model presented in previous section. There are four degrees of freedom in our model; these are number of jump processes, jump direction, jump intensity function and jump mean function. First, as in Merton 1976b, the model may include only one jump process determining the magnitude of both up and down jumps. Second, as was shown, the jump intensities may be arbitrary functions of time, $\lambda_e(t)$, including continuous distributions, or deterministic functions. Periodic functions or mixture of functions provide a particularly interesting case to study. Finally, the mean of jump magnitude can also be deterministic or random function of time, $\bar{k}(t)$. As in the case of $\lambda_e(t)$, the only needed condition is that $\bar{k}(t)$ be bounded, which leaves a wide choice of candidate functions.

This rich variety of potentials makes it possible to have boom and bust periods with different characteristics. The model proposed above only exploits the simplest characterization of boom and contraction. It is possible to describes boom and contractions with different features during different time periods, or for different securities. The following examples show additional possibilities and demonstrate the extensive applications of the theorem.

Example 1 If we set $\lambda_G(t) = \sum_p (\lambda_p^G I_p) \bar{k}^G$, $\lambda_B(t) = \sum_p (\lambda_p^B I_p) \bar{k}^B$, $k_B = k^G$ and $k_G = k^B$, then the model presented in the first section of the paper obtains.

Example 2: Merton's Model (no boom or contraction). Set $\lambda_e(t) = \lambda$ a constant , and $k_e = k$, a constant.

Example 3: Booms and contractions have deterministic (constant) jump magnitude. Set $k_G = c_G > 0$ if $0 < t < t_s$ or $k_B = c_B < 0$ if $t_s < t < T$. $\lambda_e(t)$ can be a constant, a step function with switch time at t_s, or other bounded positive functions.

Example 4: Booms and contractions have random jumps with constant mean.

Example 5: Booms have more up jumps than down jumps, but contractions only have down jumps.

Example 6: Booms only have up jumps, but contractions have more down jumps.

Summary and Conclusion

The paper proposed a new jump-diffusion model of stock price evolution, which accounts for market cycles and boom and contractionary periods. An option pricing model appropriate for this price process was derived. The model was generalized to accommodate a wide variety of possibilities.

Work in progress aims to extend the present research in a number of directions. At a theoretical level, the proposed jump-diffusion model is being altered to fit interest rate processes and to price fixed income derivatives. This work would extend the work of Das (1994) and others. Taking the proposed price process as a given, we are also exploring the problem of determining optimal consumption-portfolio decisions for a number of utility functions. This would be an extension of the work by Jarrow & Rosenfeld (1984). At a practical level, estimation is the most important means for validating the proposed model. Using various estimation techniques (Lo 1988, Lo 1986) and firm specific and index data, we are in the process of estimating the parameters of different versions of our proposed price process.

Appendix

The appendix provides the proof of main theorem in the paper. We show that equation (9) is a solution to (8). Note that the fact that (6) is a solution to (5) is a special case of the general results proven below.

Rewrite the option pricing formula (9) as

$$F(S, T - t) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} P_n(\Lambda_b(t)) P_m(\Lambda_c(t))$$

$$\times E_{n,m}\{W[V_{n,m}, T - t; K, \sigma, r]\} \tag{A.1}$$

where $V_{n,m} = SX_b(n)X_c(m)e^{-\Theta_b(t)-\Theta_c(t)}$.

Let subscripts on F and W denote partial derivatives. By differentiating (A.1), we obtain:

$$SF_s(S, T - t) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} P_n(\Lambda_b(t)) P_m(\Lambda_c(t)) \times E_{n,m}\{V_{n,m} W_S\} \tag{A.2}$$

$$S^2 F_{ss}(S, T - t) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} P_n(\Lambda_b(t)) P_m(\Lambda_c(t)) \times E_{n,m}\{V_{n,m}^2 W_{SS}\} \tag{A.3}$$

$$F_t(S, T - t) = (\sum_p \lambda_p(t))F + (\sum_p \lambda_p(t)k_p)SF_s - \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} P_n(\Lambda_b(t)) P_m(\Lambda_c(t))E_{n,m}\{W_t\}$$

$$- \lambda_b(t) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} P_n(\Lambda_b(t)) P_m(\Lambda_c(t))E_{n+1,m}\{W(V_{n+1,m}, T - t)\}$$

$$- \lambda_c(t) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} P_n(\Lambda_b(t)) P_m(\Lambda_c(t))E_{n,m+1}\{W(V_{n,m+1}, T - t)\} \tag{A.4}$$

Furthermore,

$$E_{Y_b}(F(SY_b, T - t)) = E_{Y_b}\{\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} P_n(\Lambda_b(t)) P_m(\Lambda_c(t))E_{n,m}\{W(V_{n,m}Y_b, T - t)\}\}$$

$$= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} P_n(\Lambda_b(t)) P_m(\Lambda_c(t))E_{n+1,m}\{W(V_{n+1,m}, T - t)\} \tag{A.5}$$

where the second line follows because $V_{n,m}Y_b$ and $V_{n+1,m}$ are identically distributed and the operator $E_{Y_b}E_{n,m}$ applied to a function of $Y_bX(n)$ is equivalent to the operator $E_{n+1,m}$ applied to the same function with $X_b(n+1)$ substituted for $Y_bX(n)$ (this follows from the fact that the next up jump Y_b is independent of $X(n)$). Similarly,

$$E_{Y_c}(F(SY_c, T - t)) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} P_n(\Lambda_b(t)) P_m(\Lambda_c(t))E_{n,m+1}\{W(V_{n,m+1}, T - t)\} \tag{A.6}$$

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From (A.1) to (A.6), we obtain

$$0.5\sigma^2 S^2 F_{SS}(S, T-t) + (r - \sum_p \lambda_p(t)k_p)SF_S(S, T-t) + F_t - rF(S, T-t)$$

$$= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} P_n(\Lambda_b(t))P_m(\Lambda_c(t))$$

$$E_{n,m}\{0.5\sigma^2 V_{n,m}^2 W_{SS} + rV_{n,m}W_S - W_t - rW\} - (\sum_p \lambda_p(t)k_p)SF_S + (\sum_p \lambda_p(t))F$$

$$-\lambda_b(t)E_{Y_b}(F(SY_b, T-t)) - \lambda_c(t)E_{Y_c}(F(SY_c, T-t))$$

$$= -\lambda_b(t)E_{Y_b}(F(SY_b, T-t) - F(S, T-t)) - \lambda_c(t)E_{Y_c}(F(SY_c, T-t) - F(S, T-t))$$

The last equality follows because $W(S, T-t)$ is the Black-Schole's formula, which solves:

$$0.5\sigma^2 V_{n,m}^2 W_{SS} + rV_{n,m}W_S - W_t - rW = 0 \qquad (A.7)$$

for each n and m. It follows immediately from (A.7) that F(S,T-t) satisfies equation (8). It is easy to show that the boundary conditions are satisfied as well.

Notes

[1] Of course there is a large literature on business cycles, which documents various forms of long- and short-term memory. Though the business cycles do influence the fluctuations in returns on financial assets, a discussion of the nature of this influence is beyond this paper. See Lo (1991) and Haubrich & Lo (1989) for a discussion of these and related issues.

[2] See Ball & Torous (1983), Ball & Torous (1985), Bates (1991), Bates (1995a), Bates (1995b), Bates (1994), Beckers (1981), Das (1994), Heston (1993), Ho et al. (1996), Jarrow & Rosenfeld (1984), Jones (1984), Jorion (1988), Oldfield, Rogalski & Jarrow (1977), and references there in

[3] See Lo and Wang 1995 and Pesaran and Timmermann 1995 for summary and citation to recent research in this area.

[4] Lo & Wang (1995) conjecture that jump correlation could also lead to error in the option pricing formula though they did not derive such formula. Oldfield et al. (1977) proposed and empirically tested a jump-diffusion model of asset returns with autocorrelated jumps. They found strong empirical evidence supporting their specification. However, they did not develop an option pricing formula or consider the biases which could arise from misspecification. We attempted to develop an option pricing formula for their correlated jump process but discovered that because of the correlation, no analytical solution to the partial differential equation (PDE) for the option price could be obtained. Numerical methods are needed to solve this PDE.

[5] We assume constant interest rate and volatility throughout. This provides a more streamlined setting for studying the effect of market cycles. There is no essential difficulty in expanding our analysis to include both stochastic volatility and stochastic interest rates,

[6] Again it is straight forward to add dividend rate, or more generally the 'cost of carry' to the model. We include this correction when we present the risk neutral process corresponding to the proposed asset price process.

[7] In Merton's 1976b good and bad news arrive at the same rate and are represented by draws from the tails of one distribution. We relax this assumption by permitting two different arrival rates and two distribution of jump magnitudes.

[8] Throughout we assume there exists a given probability space $(\Omega, \mathcal{F}, P)$ with respect to which $\{S\}$ is defined and that the filteratin $\{\mathcal{F}_t : 0 \leq t \leq T\}$ satisfies usual regularity conditions. We also assume there exists a a risk neutral probablilty measure $\{P^*\}$ on $(\Omega, \mathcal{F})$ such that $\{S^*\}$ is a martingale under P^*. In the next section we will be concerned with pricing a contingent claim F where $F \in L^2(\Omega, \mathcal{F}, P), \mathcal{F}_{\mathcal{T}} = \mathcal{F}$. The connection between $\{P^*\}$, a consistent price system on $L^2(\Omega, \mathcal{F}, P)$, and agent's preferences, which satisfy a convexity and continuity properties, are established in Harrison & Kreps (1979) and Chamberlain

(1988).

[9]It is a simple matter to extend our model to account for *Neutral Market* periods: In the results that follows simply set the rate of good and bad news arrivals equal. The model would reduce to that of Merton (1976*b*) if the distribution of jump magnitudes for the good and bad news are assumed to be identical.

[10]Note that no assumptions are made with respect to the distribution of inter-arrival times of these Poisson processes. It is standard practice to assume the inter-arrival times' distribution is exponential. However, for the purpose of estimation, more general Gamma distribution can also be used as in Oldfield et al. (1977).

[11]It is straight forward to relax this assumption, though one does not gain new new insights from so doing.

[12]The evidence on changes in the drift of various financial time series is presented in Perron (Econometrica, Vol. 57, 1989). We need to provide exact citation to this and other relevant literature documenting changes the expected returns over time.

[13]We used the Deleans-Dade formula, which is described in numerous text books on this subject, see for example Protter (1991).

[14] In a model with a single jump component, it is standard, following Press (1967), to assume that the jump magnitudes are log-normally distributed. As noted earlier, the log-normal assumption is in-appropriate in our model since we have restricted the supports of the distribution for the up and down jump magnitudes. Possible solutions to this problem include: 1. assume a single distribution for jump magnitudes for good and bad news but different news arrival rate, or 2. assume there is one Poisson process generating news arrival and one log-normal distribution for jump magnitudes and allow both dis-tributions to change with the phases of the market. Both solutions, would violate the optimism-pessimism market epochs our formulation attempts to capture. There are other candidate distributions for the jump magnitude (e.g., the Beta and Pareto) that have the appropriate support for our formulation. We are currently investigating alternative distributions but at the time of this writing our work is not complete. Our analysis will be completed by the time of AFA meetings.

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