

An Arbitrage-Free Approach to Quasi-Option Value¹

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In the presence of uncertainty and irreversibility, dynamic decision problems should not be solved using expected net present value analysis. The right to delay a decision can be valuable. We show that the value of this right equals Arrow and Fisher's [*Quart. J. Econom.* **88**, 312–319 (1974)] quasi-option value. In a discrete model we show how to derive quasi-option value using methods from finance. These methods yield the advantage that they permit avoidance of the common pitfall of improperly matching a riskless discount rate with a risky project. In our arbitrage-free model, use of the riskless rate is appropriate. Two main findings are presented. First, if the stochastic dynamic process underlying the problem is known, the Arrow and Fisher [*Quart. J. Econom.* **88**, 312–319 (1974)] and Henry [*Amer. Econom. Rev.* **64**, 1006–1012 (1974)] result, that improper use of net present value too often leads to early development, is correct. Second, if the process is assessed incorrectly, their result can be incorrect in the sense that net present value methods may lead to the correct outcome while the dynamic rule does not. © 1998 Academic Press

If you choose not to decide
You still have made a choice.

—Geddy Lee of Rush, *Freewill*

I. INTRODUCTION

Decision problems with stochastic, dynamic, and irreversible elements should not be treated using expected net present value (ENPV) analysis. Replacing future stochastic variables with their expected values, discounting costs and returns back to the present, and investing whenever ENPV is positive—as the rule

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prescribes—can lead to costly mistakes. It is better to solve the correct dynamic problem, accounting properly for the irreversibility, than to take advantage of the shortcut that is ENPV. This is the advice handed down by Arrow and Fisher [3] and by Henry [20]—hereafter AFH—in two similar articles addressing natural resource development problems specifically. Both articles also claim that use of ENPV will systematically favor early development. Yet ENPV is ubiquitous in environmental policy. Much of the environmental planning carried out by and on behalf of U.S. regulatory agencies from the EPA to the U.S. Army Corps of Engineers rests upon cost-benefit exercises that use ENPV analysis.³

The literature on this matter in resource economics begins with Weisbrod [31], who asserted that preservation of a natural resource confers “option value” over and above the value of development. From Weisbrod have arisen two separate strands of literature. The first, due primarily to Bohm [8], Graham [18], and Schmalensee [29], seized the phrase option value to describe a premium accruing to preservation that can be attributed to risk aversion.⁴ The second, due to AFH, is expressly independent of risk preferences. Arrow and Fisher [3] introduced the phrase “quasi-option value” (QOV) to describe the extra value that can be gained if one eschews ENPV analysis in favor of the fully dynamic alternative.

The AFH admonition against using ENPV in a dynamic and stochastic setting was not, in itself, new in 1974. It is, after all, akin to the suggestion—familiar from dynamic programming—that a dynamic problem should be solved using the appropriate dynamic techniques. Even the effects of irreversibility, a twist that AFH included, had been studied before 1974.⁵ By assuming that the development project is all or nothing, AFH added another twist, which together with irreversibility yields a scenario quite different from many dynamic programming problems in resource economics. Nonrenewable resource extraction models, for example, have as a solution a path of control decisions. With its one-time, all-or-nothing nature, the solution in the AFH model amounted to an optimal stopping rule.

For resource economists, the special appeal of the AFH articles is of course their focus on resource development problems. Bellwether articles in resource economics following the AFH tradition include, among others, Conrad [11], Fisher and Hanemann [17], and Hanemann [19]. For a 2-period model, Conrad showed that quasi-option value is equivalent to the value of information. Also working with a 2-period model, Fisher and Hanemann and Hanemann developed a definition of QOV that they called the value of information conditional upon no development in the first period. Emphasis in their work is on whether, in the second period, the decision maker knows which state has occurred.

At about the time the AFH articles appeared, Black and Scholes [7] and Merton [25] solved the problem of pricing a European option. Their work provided the analytical machinery upon which the subsequent explosive growth of the financial derivatives industry was based. In the 1980s, the usefulness of these methods for solving general investment problems was first appreciated. Prominent among the early articles exploiting the relationship between a real investment and a financial

³ For a relatively broad introduction to environmental policy from a practitioner’s viewpoint, see Arnold [1], who provides numerous examples supporting our point.

⁴ We will have nothing to say about this notion or the connected literature. See Bishop [6] and Cory and Saliba [13].

⁵ Arrow and Fisher [3] themselves cite Arrow [2] and Arrow and Kurz [4].

option are Bernanke [5], McDonald and Siegel [23, 24], and Majd and Pindyck [22]. Dixit and Pindyck's [16] recent book contains a thorough and authoritative survey of the literature and of the techniques employed there, and explains how a real investment problem can be treated as an option valuation problem.

While resource economists were developing the literature based upon AFH, a parallel literature sprang up in which the option-pricing methods were applied to resource valuation problems. Tourhino [30] appears to have originated this application. The articles by Brennan and Schwartz [9] and Paddock, Siegel, and Smith [26] are important and much-cited pieces. More recently, the financial methods were applied to natural resource problems by authors who also trace their roots to AFH. The two literatures, once largely separate, are now converging.⁶

What exactly is quasi-option value? In the present article we aim to show that the mistake one makes by inappropriately employing ENPV is the same as the mistake one makes by ignoring the right to delay a decision. The value of the mistake *is* quasi-option value, and it can be expressed

$$\text{Quasi-option value} = (\text{Value of the development opportunity including delay option} - \text{Expected net present value}).$$

Sometimes QOV = 0, either because development should never occur or because development should occur immediately, a distinction that is sometimes unclear. It will be critical to maintain another distinction, between uncertainty generated by a stochastic process whose parameters are known and correct, and an incorrect assessment of the process itself. If the process is known, the AFH insight is impeccable: the AFH rule is correct and ENPV is biased in favor of immediate development. If the process is misunderstood, the AFH insight can be wrong: ENPV might yield the correct outcome while the AFH rule does not.

The first case gives us the first theme of the article, which is that techniques from financial economics can often be used to place a value upon the right to delay an irreversible development decision. It will be seen that these techniques make expected value itself obsolete by removing all dependence of present values upon the probabilities of alternative future outcomes. We demonstrate this by developing an "arbitrage-free" alternative to net present value. The second case gives us the second theme of the article, which is that the AFH insight might sometimes be incorrect. If the level of volatility of the underlying stochastic process is assessed incorrectly, following the AFH recipe can lead to an incorrect development decision when a decision maker using ENPV would have gotten it right.

The contributions of the article are threefold. The first is a clear description of the portfolio approach to valuation. Here, we emphasize the inappropriateness of using a riskless rate to discount the costs of and returns to a risky project. Second, it is shown that if arbitrage-free methods can be used, the objective probabilities of future states become irrelevant. This has the fundamental implications that expectation in the usual sense no longer plays a role, and that the riskless discount rate may legitimately be employed. Third, we demonstrate that if the parameters (i.e., volatility) of the underlying stochastic process are assessed incorrectly, the AFH insight can lead to incorrect decisions.

⁶ Cochrane [10], Zinkhan [32], Reed [27], and Conrad [12] are just a few members of this literature.

II. EXPECTED NET PRESENT VALUE, THE DEVELOPMENT DECISION, AND QUASI-OPTION VALUE

In this section we develop a technique for calculating QOV that uses the probabilities of alternative states occurring in the future. Later it will be shown why this approach should not be used, but it is perhaps more natural than the preferred alternative, which appears in Section III.

Suppose a certain tract of wilderness land can be put to two alternative uses. It can either be preserved in its undeveloped state or it can be developed to extract some marketable resource. The developed project may be a mine, for example. In its undeveloped state, in each year t ($t = 0, 1, \dots$) the land yields a constant amenity value A from recreation uses.⁷ Letting r denote the constant risk-free interest rate and letting $R = (1 + r)$ denote the risk-free compounding factor, the value of the land if preserved is $\bar{A} = \sum_{i=0}^{\infty} A/R^i = AR/(R - 1)$.

If the land is developed at t , the stream of amenity returns is immediately and irretrievably sacrificed in return for a stream of random net revenues from the project. The developed project has a known, finite life of n years and is assumed each year to yield one unit of the marketable output with certainty. We rule out temporary shutdown and reclamation of the land after the project is completed. With the exception of a known output level, relaxing these assumptions will not substantially change our results. Instantaneous construction at period t requires a known fixed, irreversible investment of $I > 0$. Annual per-unit operating costs $c > 0$ are assumed to be constant. With this assumption the present value of the n -period stream of operating costs is given by $\sum_{i=0}^{n-1} c/R^i = c[(R^n - 1)/(R - 1)] = c\Theta$.

Output price, P_t , is the lone source of uncertainty. We assume P_t follows a stationary multiplicative random walk. That is, between t and $t + 1$ the price either rises by U percent to $P_{t+1}^+ = uP_t$ or falls by D percent to $P_{t+1}^- = dP_t$, where “+” and “-” denote the up and down states, respectively, and where $u = 1 + U$ and $d = 1 - D$.⁸ The probability of an up state at $t + 1$ is $q \in (0, 1)$. We assume that q is fixed. These conditions imply that the (P_t/P_{t-1}) are independently and identically distributed for all $t \geq 1$. Formally, we have the following definition.

DEFINITION. A price process starting at $t = 0$, denoted $PP = \{P_0, u, d, q\}$, is a binomial tree consisting of P_0 , together with the up and down increments u and d and a scalar $q \in (0, 1)$ describing the probability that the up state occurs at each t .

Because the developed project yields one unit of output in each period, price and gross revenues coincide. Given P_t , for $i \geq 0$ the expected price at $t + i$ is $E_t(P_{t+i}) = (qu + (1 - q)d)^i P_t$, and at time 0 the expected value of the stream of revenues from immediate development is $G_0(q) = \sum_{i=0}^{n-1} E_0(P_i)/R^i$. Letting $\mu = (qu + (1 - q)d)$, we have

$$G_0(q) = P_0 \left(\frac{(R^n - \mu^n)}{R^{n-1}(R - \mu)} \right) = P_0 \Omega(q),$$

⁷ This amenity value could also include nonuse values so long as these values can be counted legitimately as returns to the owner of the land. The land could be owned privately or publicly.

⁸ To prevent riskless arbitrage, u and d must satisfy the inequality $u > R > d$. Note that u and d are known constants.

where $\Omega(q)$ denotes the ratio in parentheses. A decision maker employing the ENPV rule would develop the project if $P_0\Omega(q) - (c\Theta + I) \geq \bar{A}$, and would not develop if $P_0\Omega(q) - (c\Theta + I) < \bar{A}$.

Our first objective is to derive the value of the land to its owner or to prospective buyers. Under the ENPV rule this value is

$$W_0(q) = \max[\bar{A}, P_0\Omega(q) - (c\Theta + I)]. \quad (1)$$

Suppose now that the development decision can be delayed one period, which makes use of the ENPV rule inappropriate.⁹ To calculate the value of the land in this situation, we begin at $t = 1$ and work backward. The value of the land in the up or the down state is given by

$$W_1^\pm(q) = \max[\bar{A}, P_1^\pm\Omega(q) - (c\Theta + I)]. \quad (2)$$

If the development decision is delayed, A will be collected in period 0 and the value of the land is

$$W_0^*(q) = A + \frac{qW_1^+(q) + (1 - q)W_1^-(q)}{R}. \quad (3)$$

The decision whether to develop at $t = 0$ is based upon a comparison between $W_0(q)$ and $W_0^*(q)$. If $W_0(q) > W_0^*(q)$, development should take place immediately; otherwise the optimal choice is to delay the decision. At $t = 0$, the value of the land itself, $W_0^{**}(q)$, is given by

$$W_0^{**}(q) = \max[W_0(q), W_0^*(q)].$$

Finally, the value of the right to delay the decision is the maximum of 0 and the difference between $W_0^*(q)$ and $W_0(q)$. This quantity is quasi-option value. It is here defined as

$$O^*(q) = \max[0, W_0^*(q) - W_0(q)].$$

The value of the right to delay the decision equals the cost of the mistake one commits by using ENPV in this setting. Our framework can be seen to agree with that of Fisher and Hanemann [17], though their emphasis is somewhat different. Our approach to QOV emphasizes whether development should take place immediately in a way that theirs, with its explicit conditioning of QOV on delay, does

⁹ Delay is not obligatory. Thus, the problem is akin to valuing an American call option, which grants its owner the right to exercise it at any time up to the expiration date.

not.¹⁰ A more important difference also emerges in the next section, where q drops from the analysis.

III. ARBITRAGE-FREE QUASI-OPTION VALUE

In its treatment of quasi-option value, the preceding section is in principle perfectly sound. The treatment of the dynamic and uncertain problem itself is flawed, however. The flaw inheres in the mismatch between the project's risky cash flows and the riskless rate used to discount them.

In general, projects fall into two categories: those whose risk is completely diversifiable and those whose risk is idiosyncratic, or specific to the project. If a project's risk is not completely diversifiable, one would need to employ an equilibrium model such as the capital-asset-pricing model to obtain the risk-adjusted discount factor appropriate for the particular project. This discount factor, in combination with q , could then be used to compute O^* as in the previous section.¹¹

Another alternative, which is employed in our model, is to discover the value of the land via arbitrage principles. This is done so as to include the value of the delay option. It is worth emphasizing that this approach applies only to projects with diversifiable risk. The development project we have considered falls within this category: the output price is uncertain but its riskiness is diversifiable.¹²

In order to use the arbitrage-free valuation method, a spanning condition must be satisfied: there are exactly as many states as there are assets whose prices can be discovered in the market. In the present problem the "priced" assets are the output of the project and a riskless bond. The value of the "unpriced" asset—the land—can be derived from them. In addition to the spanning condition, for the

¹⁰ Fisher and Hanemann define V^* and \hat{V} to be the expected value at $t = 0$ of the land at $t = 1$ if at $t = 1$ the decision maker does not (in the case of V^*) or does (in the case of \hat{V}) know P_1 —conditional on no development at $t = 0$. In our notation, V^* and \hat{V} correspond to the expected value at $t = 0$ of

$$W_1(q) = \max[\bar{A}, (qP_1^+ + (1 - q)P_1^-)\Omega(q) - (c\Theta + I)],$$

and

$$W_1^*(q) = \max[\bar{A}, qW_1^+(q) + (1 - q)W_1^-(q)],$$

respectively. In $W_1(q)$ the development decision at $t = 1$ cannot depend on P_1 , although in $W_1^*(q)$ it can. The mapping between our setup and Fisher and Hanemann's setup is given by

$$V^* = E_0(W_1(q)) = \frac{W_1(q)}{R} = W_0(q),$$

$$\hat{V} = E_0(W_1^*(q)) = \frac{W_1^*(q)}{R} = W_0^*(q).$$

So long as $W_0^* \geq W_0$, our O^* corresponds to their quasi-option value, V_q .

¹¹ This approach often follows Samuelson [28]. Adjusting the discount rate in this manner is difficult.

¹² This would not be true if, for example, there were no well-developed market for the output of a given project. If the project involved developing a wilderness area for recreational use, because the market for recreation services is not well developed the project's risk cannot be diversified—it is idiosyncratic. Our spanning condition is not satisfied. Even so, it is possible to draw upon financial methods. One would then have to estimate the price of the project's risk and use dynamic programming methods to value the land. See Cox and Rubinstein [15] or Dixit and Pindyck [16].

remainder of the paper we shall maintain three assumptions. First, we assume riskless arbitrage opportunities will not persist.¹³ Second, we assume no limit upon short sales. Third, there are no transactions costs. Relaxing these assumptions, familiar in financial economics, will seldom alter the results materially.

We now develop a financial approach to deriving O^* , which is driven by the absence of arbitrage opportunities. It will be seen that q is no longer needed. The parallels to the previous section are strong, and our notation reflects this, though q no longer enters the analysis. Assume as before that the decision can be delayed one period and that the parameters of the stochastic price process are known. Rather than starting with W_0 , we begin here by deriving W_0^* . We reason backward from $t = 1$, when delay is impossible. Let W_1 denote the value of the land at $t = 1$. We have

$$W_1^\pm = \max[\bar{A}, P_1^\pm \Omega - (c\Theta + I)]. \quad (4)$$

We will return shortly to derive Ω , and to show that the no-arbitrage principle ensures it is independent of q .

Now consider forming a portfolio at time $t = 0$ consisting of the land itself and a short position of quantity y of the underlying asset, say copper.¹⁴ The aim is to design the portfolio so that its value is independent of price movements at $t = 1$. The portfolio is formed by holding the land and by borrowing y units of copper, which are immediately sold at price P_0 . The money derived from the sale is invested at the riskless rate, which will yield a sure value of RyP_0 at $t = 1$. The copper is then repurchased at $t = 1$ at a cost of yuP_0 or ydP_0 , and the loan is repaid. For a given y , the value of the portfolio, Π_0 , at $t = 0$ is $\Pi_0 = W_0^* - yP_0$. Its value at $t = 1$ is

$$\Pi_1^+ = W_1^+ - yuP_0 + RA, \quad (5a)$$

and

$$\Pi_1^- = W_1^- - ydP_0 + RA \quad (5b)$$

in the up and down state, respectively. The portfolio value at $t = 1$ includes the compounded value of A . The key to risklessness is to choose y so that the portfolio has the same value in each state at $t = 1$. Setting (5a) and (5b) equal, we can solve for y as

$$y = \frac{W_1^+ - W_1^-}{P_0(u - d)}. \quad (6)$$

For this value of y , $\Pi_1 = RA + (uW_1^- - dW_1^+)/ (u - d)$ in either state and hence is riskless.

¹³ An arbitrage opportunity is a situation in which one can form a costless portfolio that has strictly positive future cash flows.

¹⁴ This is a *hedge portfolio*. We could alternatively have formed a *replicating portfolio*, consisting of the riskless bond and the underlying asset, to replicate the payoffs to the land. The results would be identical. In either case, the assumptions of no limit on short sales and no transactions costs are invoked.

To solve for W_0^* , we invoke the requirement that the portfolio, being riskless, must yield precisely the riskless rate.¹⁵ It is in this requirement that the price of the riskless bond—the other spanning asset—is put to use. Letting $S = (R - 1)yP_0$ denote the cost of holding the short position (the trader on the other side of the transaction requires at least this payment), the gain to holding the portfolio is $\Pi_1 - \Pi_0 - S$. This quantity must equal $(R - 1)\Pi_0$, the riskless return on holding Π_0 for one period, hence $\Pi_1 - \Pi_0 - S = (R - 1)\Pi_0$. Using the expression for y , and a bit of algebra, this can be manipulated to yield

$$RW_0^* = RA + \frac{uW_1^- - dW_1^+}{u - d} + \frac{W_1^+ - W_1^-}{u - d}.$$

For later reference, note that $\Pi_0 = (\Pi_1 - S)/R$. Finally, we can write the value of the land, allowing for delay, as

$$\begin{aligned} W_0^* &= A + \frac{1}{R} \left[\frac{(1 - d)W_1^+ - (1 - u)W_1^-}{u - d} \right] \\ &= A + \frac{pW_1^+ + (1 - p)W_1^-}{R}, \end{aligned} \quad (7)$$

where $p = (1 - d)/(u - d)$. Remarkably, given P_0 the probability q of an up state, and by extension the expected price, is irrelevant to the value of the land.¹⁶ Equivalently stated, the no-arbitrage principle implies that risk preferences play no role in valuing the land by this method.

What is one to make of p ? It appears now to play the role of q , which suggests that p is like a probability, though it does not describe the likelihood of any actual event. Cox and Ross [14] called p a “risk-neutral probability,” because in conjunction with R , as in (7), one obtains the value W_0^* that a rational decision maker would place on the portfolio. The phrase “risk-neutral” is unfortunate insofar as it suggests the result is valid only if the decision maker is risk neutral. On the contrary, the replication method works regardless of risk preferences. The absence of arbitrage forces risk-averse investors and risk-loving investors to agree on the correct value of W_0^* .

The value of Ω is also calculated using p rather than q . It is not difficult to see why this is true. In the last year of the project’s life, its value can be derived by forming a portfolio just as before, assigning “probability” p to an up state and discounting using R . Working backward, we see that the arbitrage-free value at

¹⁵ This is the point at which our three assumptions come into play. The no-arbitrage condition is used, for if the portfolio does not yield the riskless rate our decision maker would be the source of arbitrage profits.

¹⁶ This is also true in the continuous-time version of our model. In a wonderful understatement, when describing the differential equation at the heart of Black and Scholes’s [7] continuous-time option-pricing model, to which our treatment can be traced, Hull [21, p. 239, emphasis added] writes, “The Black–Scholes differential equation would not be independent of risk preferences if it involved the expected return on the stock, μ . This is because the value of μ does depend on risk preferences. The higher the level of risk aversion by investors, the higher μ will be for any given stock. *It is fortunate that μ happens to drop out in the derivation of the equation.*” Fortunate indeed. The result is the linchpin—and the genius—of the method.

period t of the stochastic returns to the project developed at t is

$$G_t = P_t \sum_{i=0}^{n-1} \frac{\varphi^i}{R^i} = P_t \Omega,$$

where $\varphi = pu + (1 - p)d$, $\Omega = (R^n - \varphi^n)/(R^{n-1}(R - \varphi))$, and n is the life of the project. But it is easy to check that given our definition of p , $\varphi \equiv 1$.¹⁷

The remaining elements of the model may now be derived. The counterpart to $W_0(q)$ from the previous section is

$$W_0 = \max[\bar{A}, P_0 \Omega - (c\Theta + I)].$$

This is not an expected value, so rather than ENPV we will call it ANPV: arbitrage-free net present value. The value of the land is

$$W_0^{**} = \max[W_0, W_0^*].$$

Quasi-option value, which is the value of the right to delay the decision, is given by

$$O^* = \max[0, W_0^* - W_0].$$

The connection between O^* and the development decision is strong. If $O^* = 0$ and $W_0 > \bar{A}$, one should develop immediately.¹⁸

An example will help to fix ideas. Suppose the parameters in the model are as in Table I. Figure 1 contains the four curves of interest— W_0 , W_0^* , W_0^{**} , and O^* —as a function of P_0 . The other parameters are fixed at the values given in Table I, so $\bar{A} = 825$. To begin, observe that there are two kinks in W_0^* and one in W_0 and denote them P'_0 , P''_0 , and P'''_0 in the figure. The interpretations to be placed on them are as follows.

P'_0 : This is the price corresponding to the first kink in the W_0^* curve. If $P_0 < P'_0$, a decision maker who defers the decision would not develop at $t = 1$ even in the up state. Thus, if $P_0 < P'_0$ the project should never be developed.

¹⁷ Thus, our approach to valuing a stochastic stream of returns amounts to assuming the price remains fixed at P_0 in the future. This is a subtle but important point. Rather than presenting this argument for deriving the arbitrage-free G_t , we could simply have assumed that at the moment of development the owner could contract with a buyer to sell the entire stream of production at price P_t . The results would have been identical to what we obtain. This is because assuming we can create the riskless portfolio is equivalent to assuming that the riskiness of the project is completely diversifiable—which it must be if such a forward contract is available.

¹⁸ It is well known that an American call option on an asset that pays no dividends should never be exercised early (Merton [25]). This result does not apply in the present problem. The output of the developed project is like a dividend payment. Thus, early exercise may be optimal, and is optimal precisely when $O^* = 0$ and $W_0 > \bar{A}$.

TABLE I
Example Parameter Values

Parameter	R	n	u	d	q	I	A	c
Value	1.1	10	1.3	0.7	0.60	1000	75	50

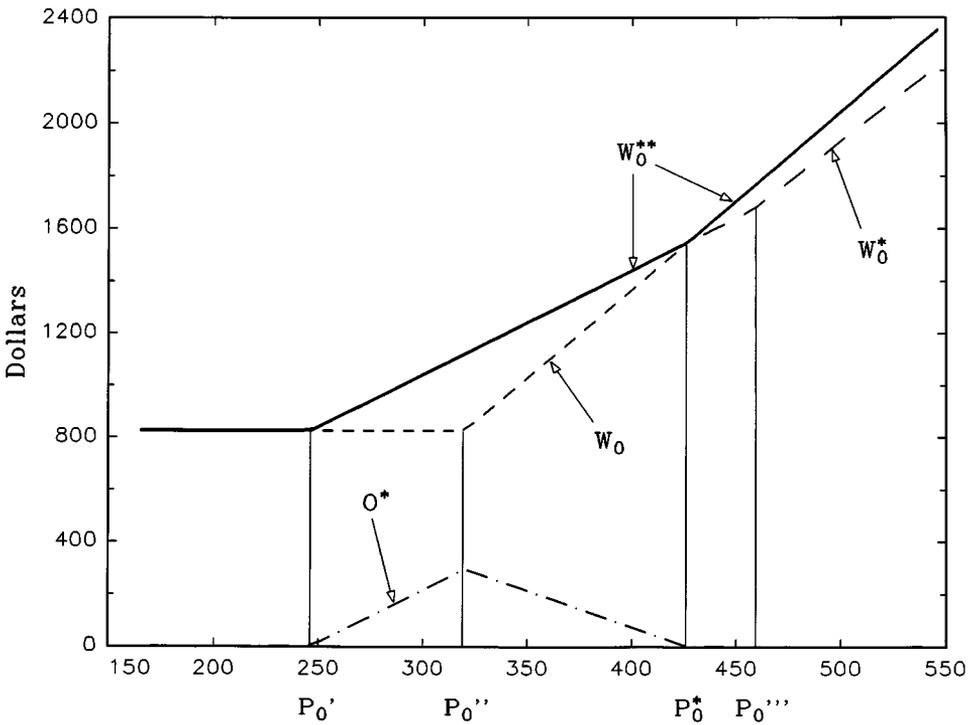


FIG. 1. W_0 , W_0^* , W_0^{**} , and O^* as a function of P_0 .

P_0' : This is the price corresponding to the kink in the W_0 curve. If $P_0 > P_0'$, a decision maker employing the ANPV rule would develop immediately.

P_0'' : This is the price corresponding to the second kink in the W_0^* curve. If $P_0 > P_0''$, a decision maker who defers the decision would develop at $t = 1$ even in the down state.

One other price is important. Indeed, it turns out to be the most important for our story.

P_0^* : This is the price at which W_0 and W_0^* intersect. If $P_0 > P_0^*$, the optimal decision is to develop the project immediately.

It turns out that P_0''' is never interesting. It is straightforward to show that $P_0''' > P_0^*$ always holds: if development should occur at $t = 1$ even in the down state, then development should not be delayed.

Consider now the O^* curve. We see that $O^* = 0$ in two regions: where $P_0 < P_0'$ and where $P_0 > P_0^*$. In the first, development should never take place. The right to wait until tomorrow to decide whether to develop has no value. In the second, development should take place immediately. Once again the right to wait until tomorrow has no value.¹⁹ For $P_0 \in (P_0', P_0^*)$, the story is very different. Here the right to wait is valuable. The most important range of initial prices, at least in the

¹⁹ This possibility appears to be what Arrow and Fisher [3, p. 319] meant when they wrote, "Just because an action is irreversible does not mean that it should not be undertaken."

context of the AFH insight, is (P_0'', P_0^*) . For prices in this interval, ANPV analysis leads one to develop immediately though the best decision is to wait and to develop at $t = 1$ only in the up state.

The price P_0^* plays the role of a trigger: for a given set of parameters, if the current price exceeds P_0^* development should occur immediately. In this case, employing ANPV leads to the correct decision to develop immediately. Of course, in practice one must calculate the portfolio to derive P_0^* , so it is never safe to use the ANPV rule alone.

Given its function as a trigger value, we wish first to discover how P_0^* responds to changes in the parameter values. We show that as A or I grow, the threshold development price rises, making immediate development less likely. On the other hand, as the life of the project (n) increases, the threshold development price falls.

PROPOSITION 1. *For any given parameter vector (R, n, u, d, q, I, A, c) , P_0^* is (i) increasing in A , (ii) increasing in I , and (iii) decreasing in n .*

Proof. We begin by deriving an analytical expression for P_0^* . Note first that, because $P_0''' > P_0^*$, if development is delayed it must be true that in the relevant region $W_1^- = \bar{A}$. The expression for W_0^* thus becomes

$$W_0^* = \frac{p(uP_0\Omega - (c\Theta + I)) + (1 - p)\bar{A}}{R} + A. \tag{8}$$

Using the definition of W_0 , we can solve for P_0^* by setting $W_0 = W_0^*$, which yields

$$P_0^* = \frac{1}{\Omega} \left(\frac{(R - p)(c\Theta + I) + (1 - p)\bar{A} + RA}{R - pu} \right). \tag{9}$$

The denominator of the rightmost term must be positive. To see this, note that $pu = (u - ud)/(u - d) < 1$, and $R > 1$ by assumption. Thus, the proof of (i) and (ii) follows without difficulty from partial differentiation of P_0^* with respect to A and I .

Result (iii) is a bit more subtle. It suffices to show that $P_0^*(n + 1)/P_0^*(n) < 1$. By definition, $\Omega(n + 1) > \Omega(n)$ for any $n \geq 1$. Substituting $\Omega = \Theta$ (which is true because $\varphi = 1$), we have

$$\begin{aligned} & \frac{P_0^*(n)}{P_0^*(n + 1)} \\ &= \frac{\Omega(n)}{\Omega(n + 1)} \left(\frac{((R - p)(c\Omega(n + 1) + I) + (1 - p)\bar{A} + RA)/(R - pu)}{((R - p)(c\Omega(n) + I) + (1 - p)\bar{A} + RA)/(R - pu)} \right). \end{aligned} \tag{10}$$

Combining terms, let $b = ((R - p)I + (1 - p)\bar{A} + RA)$. The ratio in (10) may now be written

$$\begin{aligned} \frac{P_0^*(n)}{P_0^*(n + 1)} &= \frac{\Omega(n)}{\Omega(n + 1)} \left(\frac{c(R - p)\Omega(n + 1) + b}{c(R - p)\Omega(n) + b} \right) \\ &= \frac{c(R - p)\Omega(n)\Omega(n + 1) + b\Omega(n)}{c(R - p)\Omega(n)\Omega(n + 1) + b\Omega(n + 1)} < 1. \end{aligned}$$

The last inequality holds because $\Omega(n+1) > \Omega(n)$. This completes the proof of the proposition. ■

Everything to this point was driven by the irreversibility of the development decision, together with uncertainty about the future returns to developing the land. To complete this section and to set the stage for the next, we now turn to a deeper question: what if the *level* of uncertainty increases?²⁰ We examine the effect upon P_0^* and O^* of a mean-preserving increase in the volatility of the price process, which in this binomial setting takes the form of an increase in the spread of the price realizations in each period.

The relationship between u , d , and q was unspecified to this point. Assume there is a continuous-time stochastic process with drift μ and volatility (or standard deviation) σ that determines the evolution of the price process. For our binomial model to be consistent with the widely used Brownian price process, the following must hold:²¹

$$u = e^{\sigma\sqrt{\Delta t}}, \quad (11a)$$

$$d = \frac{1}{u}, \quad (11b)$$

and

$$q = \frac{1}{2} + \frac{1}{2} \left(\frac{\mu}{\sigma} \right) \sqrt{\Delta t}. \quad (11c)$$

For simplicity we let $\Delta t = 1$.

This formulation, which links u and d to a single parameter that captures the level of variability of the process, will prove useful in the sequel. So long as u , d , and q are computed according to Eqs. (11), an increase in σ with μ fixed is a mean-preserving spread in the process.

DEFINITION. Given a price process PP , where u , d , and q are defined by Eqs. (11), together with a pair (μ, σ) , a mean-preserving spread is an increase in σ .

Note that because φ remains constant, a mean-preserving spread leaves Ω and W_0 unchanged. We are now prepared to show that P_0^* , the threshold price, increases in σ . Thus, the greater the price dispersion, the greater the inclination to delay development.

PROPOSITION 2. *A mean-preserving spread in the price process PP causes an increase in P_0^* . That is, $\partial P_0^*/\partial \sigma > 0$.*

Proof. The definition of d implies that $p = (u + 1)^{-1}$. Thus, $dp/du = -1/(u + 1)^2 < 0$. Let $\delta = (R - p)(c\Theta + I) + (1 - p)\bar{A} + RA > 0$ denote the numer-

²⁰ This question was investigated by Hanemann [19], who found that his quasi-option value increases in response to an increase in the uncertainty of future realizations.

²¹ That is, $dP = \mu P dt + \sigma P dz$, where z is a standard Weiner process. See Cox and Rubinstein [15] for a discussion of the close connection between the discrete- and continuous-time versions of the problem. In particular, as $\Delta t \rightarrow 0$, the discrete-time version approaches the continuous-time version. Furthermore, the discrete-time μ that denoted the mean of a discrete process in the previous section, approaches its value here as an instantaneous mean.

ator of the term in parentheses in Eq. (9). Then

$$\begin{aligned} \frac{\partial P^*}{\partial \sigma} &= \frac{\partial P^*}{\partial u} \frac{du}{d\sigma} \\ &= \frac{1}{\Omega} \left(\frac{\delta(p + u(dp/du)) - (R - pu)(c\Theta + I + \bar{A})(dp/du)}{(R - pu)^2} \right) e^\sigma. \end{aligned}$$

The term multiplying δ , $p + u(dp/du)$, can be reduced to $1/(1 + u)^2 > 0$. Because $dp/du < 0$, then, the term in parentheses is positive. Thus, $\partial P^*/\partial \sigma > 0$, which was to be proved. ■

We now show that an increase in σ leads unambiguously to an increase in quasi-option value. Intuitively, this is due to the ability to avoid an ever less attractive outcome by waiting.

PROPOSITION 3. *Quasi-option value O^* is nondecreasing in response to a mean-preserving spread in the price process. If $O^* > 0$, it is strictly increasing in response to a mean-preserving spread.*

Proof. Consider first the case in which $O^* = 0$, so that $P_0 < P_0^*$. Obviously, by its definition, O^* cannot decrease. Now suppose that $O^* > 0$, which means that $P'_0 < P_0 < P_0^*$. Because φ always equals 1, Ω is constant as σ changes and $W_0 = \max[\bar{A}, \Omega P_0 - (c\Theta + I)]$ is invariant to changes in σ . Thus, O^* increases precisely when W_0^* increases. But in the relevant region W_0^* is given by Eq. (8). Thus, the response of O^* to an increase in σ is given by the following derivative,

$$\begin{aligned} \frac{\partial O^*}{\partial \sigma} &= \frac{\partial W_0^*}{\partial \sigma} = \frac{\partial W_0^*}{\partial u} \frac{du}{d\sigma} \\ &= \frac{1}{R} \left(P_0 \Omega \left(p + u \frac{dp}{du} \right) - \frac{dp}{du} (c\Theta + I + \bar{A}) \right) \frac{du}{d\sigma}. \end{aligned}$$

But from the proof of Proposition 2, we know that $(p + u(dp/du)) > 0$ and that $dp/du < 0$. Thus, the derivative $\partial O^*/\partial \sigma$ must be strictly positive, and the proof is complete. ■

Figure 2 illustrates the effect of changes in σ on W_0 , W_0^* , W_0^{**} , and O^* . The four panels of the figure differ only in the value of P_0 . In Fig. 2(a), where $P_0 = 150$, for σ smaller than about 0.75, $O^* = 0$. At such a low price, if σ is small development should never occur. If σ is above 0.75, though, waiting becomes valuable as P_1^+ becomes large enough to drive W_1^+ above \bar{A} . Figure 2(b) is similar, except there is no value of σ for which $O^* = 0$. Notice that O^* is also everywhere greater than it was in Fig. 2(a). In Fig. 2(c), where $P_0 = 450$, we once again find that $O^* = 0$ for small σ . Contrary to Fig. 2(a), here $O^* = 0$ does not mean that development should never occur but that it should occur immediately.

This result linking σ to the value of the right to delay is important in many real-world resource development problems. The perceived volatility of future returns can be large, and the larger the volatility the greater the value of waiting.

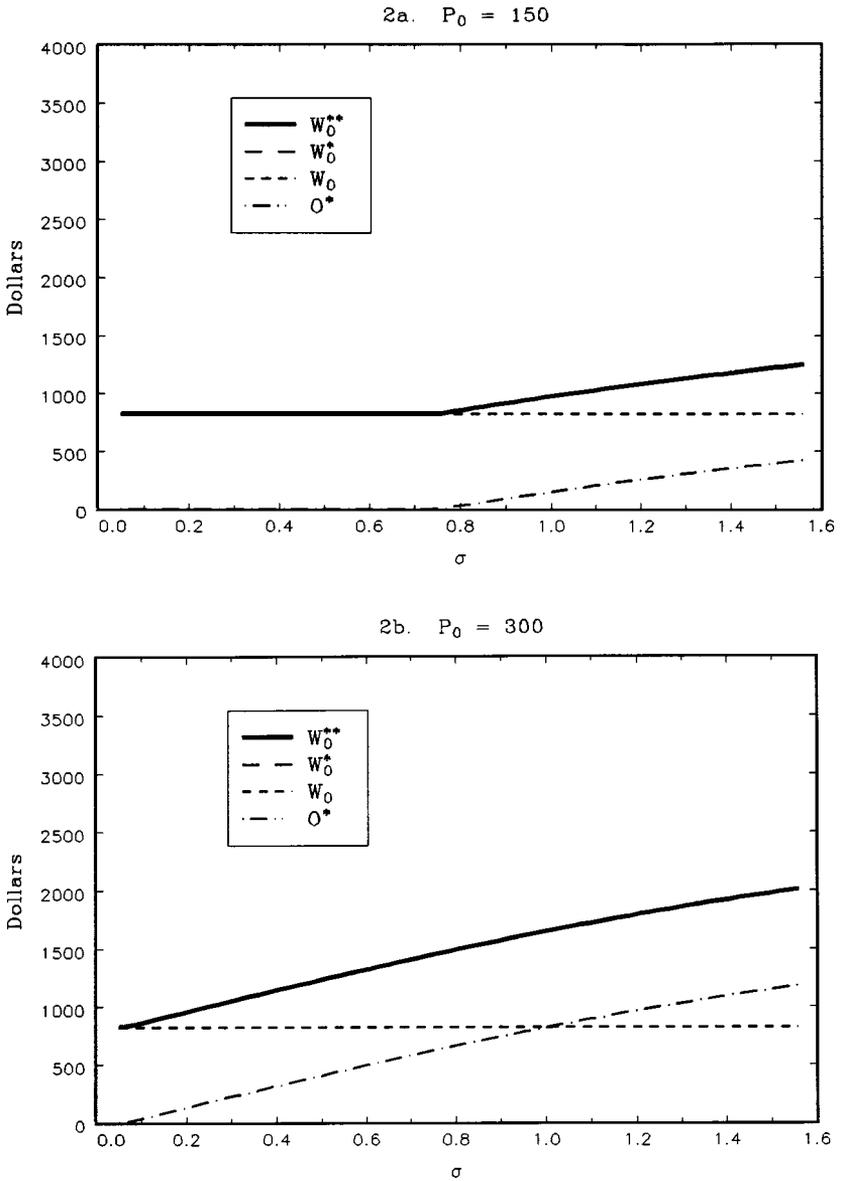


FIG. 2. W_0 , W_0^* , W_0^{**} , and O^* as a function of σ . $P_0 = 150, 300, 450$, and 600 .

IV. QUASI-OPTION VALUE WITH INCORRECT ASSESSMENT OF σ

To this point, it was assumed that the parameters of the underlying stochastic process were correctly assessed. In this setting there is nothing more one can know. The stochastic process is all. It is now time to relax this assumption and to suppose that though our decision maker has obtained an assessment of σ , denoted σ_a for "assessment," his or her assessment may be incorrect. Note that though the decision maker believes σ_a to be correct and behaves accordingly, the passage of

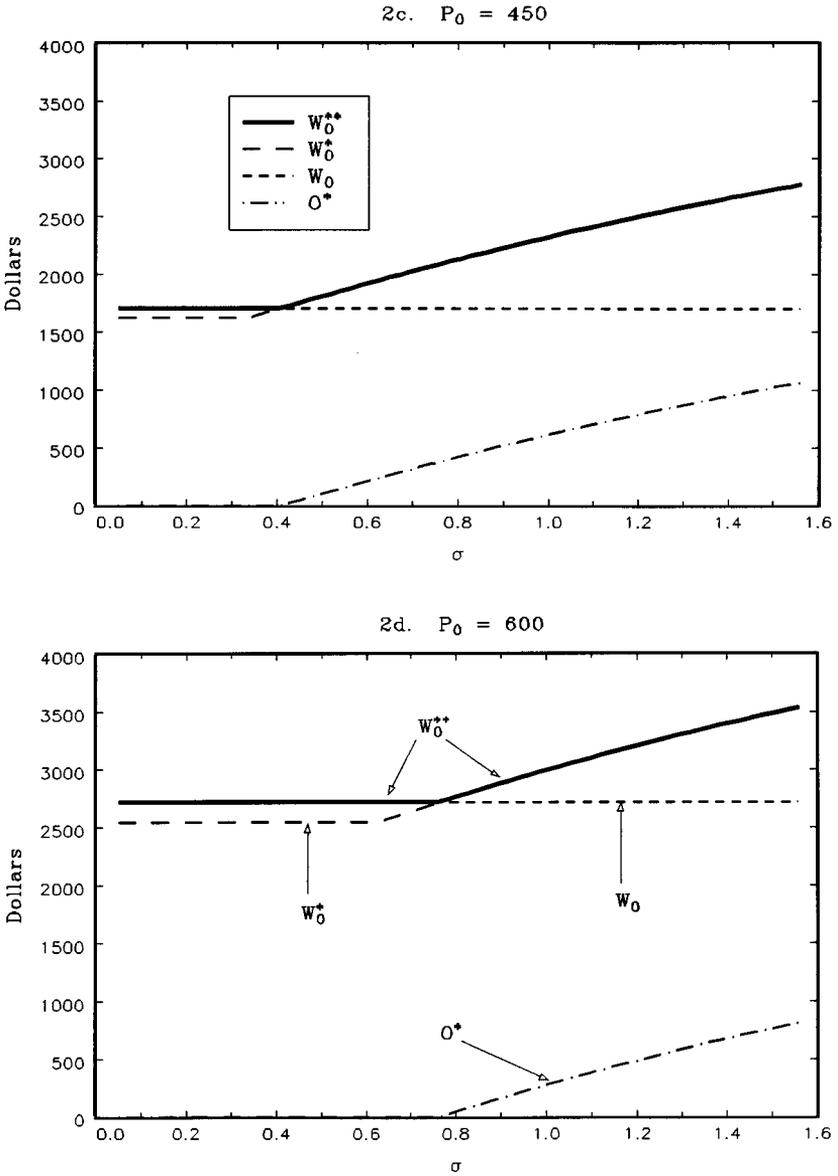


FIG. 2. (Continued)

time may prove that σ_a is incorrect. Our aim is to show how fragile the AFH recommendation can be in this situation. All that is required is for the decision maker to be wrong about σ .²²

Suppose the price process is described by Eqs. (11). What, if anything, changes if σ_a is in fact incorrect? The decision maker must calculate the value of the land,

²² We do not imply σ is random. One could specify a distribution for σ . In this instance the spanning condition would require the existence of an asset for hedging the additional source of uncertainty. In practice no such asset exists.

and in the belief that σ is correct, this decision will be made using the method of the previous section. Is this scenario far-fetched? We believe it is not. Calculating the volatility of a stochastic process is usually difficult and sensitive. Yet professional traders routinely form portfolios using methods like ours. Sometimes they later learn that their assessment of σ was incorrect, as our decision maker may do.

In the previous section the portfolio was hypothetical, formed only to price the land. The calculation also yielded the optimal y , along with W_0^{**} and O^* . In practice, it is at the decision maker's discretion whether to protect against risk by forming this portfolio or to hold a naked position. In the remainder of this section we will assume the portfolio is formed if the decision is delayed when development may occur (that is, if $P_0 \in [P_0', P_0^*]$) and that the land is not sold.

We wish to understand the effect upon the valuation problem and the development decision if $\sigma \neq \sigma_a$. The section possesses two main thrusts. First, we show how to calculate the effect of a mistake in specifying the stochastic process using the portfolio methods previously described. Second and more important, we show that it is possible in this case that following the AFH program (develop immediately only if $W_0 > W_0^*$) can lead to a *worse* outcome than following the simple ANPV rule (develop immediately only if $W_0 > \bar{A}$).

If $\sigma \neq \sigma_a$, the effect is felt in two ways. First, because σ_a is wrong our decision maker will choose an incorrect short position and the portfolio will be imperfect. The *financial loss* due to being wrong about σ is the value today of the difference between the portfolio value that would be achieved if σ were known, and the portfolio value that is achieved by acting as if σ_a were correct. In both cases we evaluate the portfolio using the values of P_1^\pm that obtain under the correct σ . Second, if σ_a is wrong there may be an error in the development decision itself. This mistake could go either way: the project may be developed immediately when it should be delayed, or it may be delayed when it should be developed immediately. We will call this the *real loss* due to being wrong about σ . The total loss from being wrong is the sum of the two quantities.

Recall that the portfolio consists of the land and a short position in copper. We saw that this portfolio's value at $t = 0$ equals the value of the portfolio at $t = 1$ less the cost of the short position, discounted 1 period: $\Pi_0 = (\Pi_1 - S)/R$. Now, let $u_a = e^{\sigma_a}$ and $d_a = 1/u_a$ be the up and down increments as a function of σ_a , and let y_a be the short position in the portfolio, calculated as in Eq. (6). This portfolio differs from the correct one only in the size of the short position in copper. At $t = 1$, the portfolio's value in the up and down states will be given by Eq. (5) using y_a and the correct P_1^\pm based upon σ . Denote these values $\Pi_{1,a}^\pm$. Though the decision maker is unaware of it, this portfolio is not riskless: $\Pi_{1,a}^+ \neq \Pi_{1,a}^-$. The decision maker discounts the portfolio using R and assigns "probabilities" p and $1 - p$: $\Pi_{1,a} = p\Pi_{1,a}^+ + (1 - p)\Pi_{1,a}^-$. Thus, using $S_a = (R - 1)y_a P_0$,

$$\Pi_{0,d} = \frac{\Pi_{1,a} - S_a}{R}.$$

If the portfolio is formed, the financial loss due to a mistaken σ_a is the difference,

$$\begin{aligned} L_f(\sigma, \sigma_a) &= \Pi_0 - \Pi_{0,d} \\ &= \frac{p(y_a - y)P_1^+ + (1 - p)(y_a - y)P_1^- - (R - 1)(y - y_a)P_0}{R}. \end{aligned}$$

Note that if this quantity is positive, the mistake leads to a gain; if it is negative, the mistake leads to a loss. If development occurs immediately or if the decision maker expects never to develop, there will be no portfolio. In this case $L_f = 0$.

The financial loss is only part of the story. Potentially more important is the possibility that if σ_a is wrong an incorrect development decision might be chosen. For a given set of parameter values, we can define the threshold value σ^* at which O^* becomes positive,

$$\sigma^* = \inf\{\sigma : O^* > 0\}.$$

This threshold value plays a crucial role in determining the real loss due to a mistaken assessment of σ_a . We observe without proof that if P_0 is finite, then σ^* is finite, and for intuition we direct the reader to Fig. 2. There are two possibilities: either $ANPV = \bar{A}$ (as in Fig. 2a) or $ANPV > \bar{A}$ (as in Figs. 2(c) and 2(d)). If $ANPV = \bar{A}$, $\sigma \leq \sigma^*$ means that development should never take place, and $\sigma > \sigma^*$ means that development should be delayed and the portfolio should be formed. In the latter case, development will occur at $t = 1$ in the up state, and the portfolio is formed in an attempt to remove the riskiness of owning the land. If $ANPV > \bar{A}$, $\sigma \leq \sigma^*$ means that development should take place immediately, and $\sigma > \sigma^*$ once again means that development should be delayed.

Based upon σ^* , for each of the two situations just described ($ANPV = \bar{A}$ or $ANPV > \bar{A}$) we can divide (σ, σ_a) -space into four regions. Table II describes the possible relationships between the decision made using σ and the decision made using σ_a if $ANPV = \bar{A}$, when an ANPV decision maker *would not* develop at $t = 0$.²³

Note that here the decision maker will never develop the project immediately. No matter what σ_a happens to be, this is the correct decision and thus there is no real loss. If $ANPV > \bar{A}$, though, this is no longer true. Table III describes the relationships between the decision made using σ and the decision made using σ_a if $ANPV > \bar{A}$, when an ANPV decision maker *would* develop at $t = 0$.

Now it is possible that a mistaken σ_a can lead to an incorrect development decision. In Region II, development occurs immediately when it should be delayed. The real cost of this error is simply foregone quasi-option value, O^* . Because no portfolio is formed, there is no financial loss. In Region III, development is delayed when it should occur immediately. The real cost of this error is $W_0^* - W_0$, a negative number because in this region $W_0 > W_0^*$.

²³ In the table, the phrase “walk away” indicates that the decision maker decides development will never be profitable, so does not develop at $t = 0$ and does not form a portfolio.

TABLE II
Decisions if $ANPV = \bar{A}$

	$\sigma \leq \sigma^*$	$\sigma > \sigma^*$
$\sigma_a \leq \sigma^*$	Region I: Correctly walk away; form no portfolio	Region II: Incorrectly walk away; should form a portfolio
$\sigma_a > \sigma^*$	Region III: Incorrectly form a portfolio; should walk away	Region IV: Correctly form a portfolio

TABLE III
Decisions if $ANPV > \bar{A}$

	$\sigma \leq \sigma^*$	$\sigma > \sigma^*$
$\sigma_a \leq \sigma^*$	Region I: Correctly develop immediately; form no portfolio	Region II: Incorrectly develop immediately; should delay, form portfolio
$\sigma_a > \sigma^*$	Region III: Incorrectly delay; should develop immediately	Region IV: Correctly delay, form portfolio

The real loss suffered due to a mistaken σ is 0 if the correct development decision (either delay or develop) is made. However, as function of σ and σ_a , real loss can be expressed as follows,

$$L_r(\sigma, \sigma_a) = \begin{cases} 0 & \text{if } ANPV = \bar{A}; \\ 0 & \text{if } ANPV > \bar{A} \text{ and Region I or IV;} \\ -O^* & \text{if } ANPV > \bar{A} \text{ and Region II;} \\ W_0^* - W_0 & \text{if } ANPV > \bar{A} \text{ and Region III.} \end{cases}$$

Total loss due to a mistaken σ is the sum of L_f and L_r , which we may express

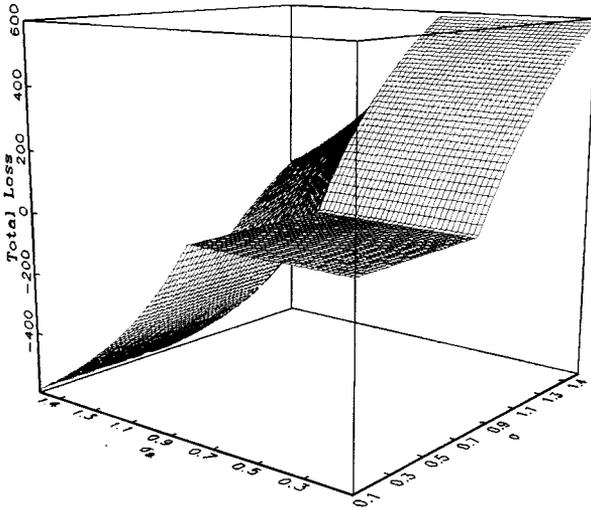
$$L(\sigma, \sigma_a) = L_f + L_r = \begin{cases} L_f & \text{if } ANPV = \bar{A}; \\ L_f & \text{if } ANPV > \bar{A} \text{ and Region I or IV;} \\ L_f - O^* & \text{if } ANPV > \bar{A} \text{ and Region II;} \\ L_f + W_0^* - W_0 & \text{if } ANPV > \bar{A} \text{ and Region III.} \end{cases}$$

The three-dimensional surface of $L(\sigma, \sigma_a)$ is depicted in Figure 3 for four different values of P_0 , the initial price.

From the figure, note that the financial loss, L_f , can be positive. In Fig. 3(a), for example, where P_0 is such that $ANPV = \bar{A}$, $L_f > 0$ in Region II. One might expect that being wrong in an arbitrage-free setting is always costly. Not so. In Fig. 3(a), if $\sigma = 1.2$ and $\sigma_a = 0.5$, for example, our decision maker has formed a portfolio which, by sheer luck, yields a financial gain. Because $\sigma_a < \sigma$, the short position is too small: $y_a < y$. Because of this, in the down state at $t = 1$ the short-sale loss is smaller than it would have been using y . The value of the land in the up state, meanwhile, is greater than anticipated. The development decision is correctly delayed (as it always is when $ANPV = \bar{A}$), so this financial gain is not offset by a real loss.

In Fig. 3(c), $L < 0$ in region II, but $L > 0$ in Region III. In both cases the wrong development decision is taken. In Region II, σ_a is small enough that it leads incorrectly to immediate development. The realized loss here is O^* . This is the same mistake an ANPV decision maker would make by ignoring the value of the delay option. In Region III, because $\sigma < \sigma^*$, the correct decision would have been to develop the project immediately. Our decision maker delays, which yields a real

3a. $P_0 = 150$



3b. $P_0 = 300$

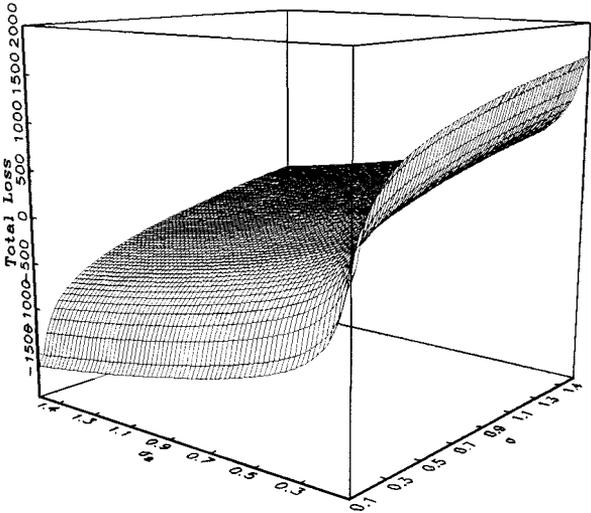


FIG. 3. Loss due to mistaken σ_a , as a function of σ and σ_a . $P_0 = 150, 300, 450,$ and 650 .

loss of $W_0^* - W_0 < 0$. The financial position, however, yields a gain that more than offsets this loss. The end result is that $L > 0$, representing mistakes that do not cause an incorrect development decision but are profitable financially.

Finally, all that remains is to support the claim that if $\sigma_a \neq \sigma$, the ANPV rule may yield the correct development decision though the AFH rule yields the incorrect development decision. The illustration of this claim is found in Fig. 3(d), in which $P_0 = 650$. Here, we have $\sigma^* = 0.85$. The result we seek is achieved when $\sigma = 0.5$ and $\sigma_a = 0.87$. Because $ANPV > \bar{A}$, the wrong development decision is

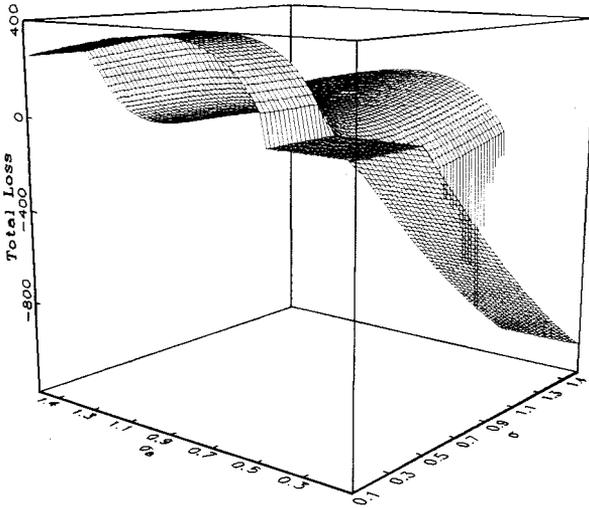
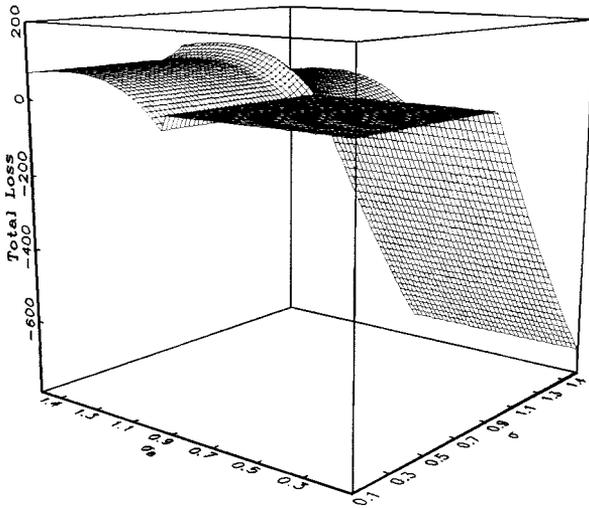
3c. $P_0 = 450$ 3d. $P_0 = 650$ 

FIG. 3. (Continued)

made in Regions II and III. In Region III, development is delayed when it should not be *and the ANPV rule would have led correctly to immediate development*. In many cases the cost of the real mistake is offset by a gain in the financial side. In some cases, however, the development decision is wrong and the value achieved is lower under the AFH rule than it would have been under ANPV.

PROPOSITION 4. *There exist cases in which the AFH rule leads to delay when development should occur immediately. In some of these, the period-0 value of the returns to the land—including the outcome of the short position—is negative.*

Proof. The first statement is true if $ANPV > \bar{A}$ and $\sigma < \sigma^* < \sigma_a$. In order to prove the second, we need only present one situation in which it is true. Let the parameters P_0 , R , n , I , A , and c take values 280, 1.1, 10, 1000, 75, and 50, respectively. Let $\sigma = 0.5$ and let $\sigma_a = 0.87$. In this situation we have (after rounding),

$$\begin{aligned} W_0 &= 3055.4, & W_0^* &= 2852.6 \\ L_f &= 148.9 & L_r &= -202.8, & L &= -53.9. \end{aligned}$$

It is evident that the correct decision would be to develop the project immediately, which an ANPV decision maker would do. The decision maker using the AFH rule, however, would incorrectly delay development. This would lead to a real loss of $L_r = -202.8$. The financial position taken to remove the perceived riskiness of this project yields a positive return of $L_f(\sigma_a) = 148.9$. The sum of these, however, is negative. This completes the proof of the proposition. ■

Figures 3(c) and 3(d) suggest the following caution. If the ANPV calculation recommends immediate development, it appears that less harm is done if the assessment of σ is too high than if it is too low. Of course, which of the different possibilities is most common is an empirical matter. But although this section has shown that sometimes the dynamic AFH rule (develop if $W_0 > W_0^*$) is faulty, we may also extract a final recommendation that is certainly in the same spirit as AFH: be conservative. The most costly mistakes tend to be in the direction of premature development.

V. CONCLUDING REMARKS

The option to delay an irreversible decision can be valuable. This is true in natural resources as in general investment problems. Ignoring this option, which one does when the net-present-value rule is used, can be costly. How costly? We have shown how to calculate the cost, QOV, for a simple model using arbitrage-free methods. If the parameters of the stochastic process are correctly known, in our setting the AFH result holds true. One should not use a net-present-value rule. If the parameters are incorrectly known, however, things are not so clear. In some situations the rule yields the correct decision (which is the decision a person who knows the process correctly would make) while the dynamic AFH rule yields the incorrect decision.

We also showed that QOV grows as the volatility of the process grows. For a development problem whose source of uncertainty has low volatility, then, the potential cost of improper early development is low. At the same time, as Figs. 3(c) and 3(d) indicate, the worst mistake one can make involves an underassessment of volatility and resulting premature development.

Our simple, discrete model could be extended in many directions. The delay option could be extended to an arbitrary number of periods. The amenity benefits, A , could be made stochastic, although one may not be able to use arbitrage-free methods in this case (see footnote 12). A number of additional options could be introduced. These might include the option to shut down the project, to expand it, to speed up production (thereby shortening its life), or to reclaim the land after the project is over.

These extensions are all feasible in a discrete model. They would be more easily accomplished in a continuous-time model, though, because the discrete version quickly becomes unwieldy. As noted earlier, we used a discrete model primarily because we feel it is a better vehicle for describing how arbitrage-free methods from financial economics can be employed to value natural resources. The task of exploring the effect of other options is deferred to future work.

For purposes of creating realistic models of development problems, particularly if these models are to be employed empirically using discrete data, the subtlety and difficulty of continuous-time models should not be underestimated. One appeal of the Black–Scholes model is that it has an analytical solution. Seemingly straightforward extensions can erase this advantage. Only very seldom can a real-world problem be squeezed into the pure Black–Scholes framework. The most exciting current work on resource development decision making, it seems to us, consists in stretching the replication methods to cover more realistic cases.

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