Insurance and reinsurance contracts as complex derivatives: Application to multiple peril policies

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Multiple peril insurance schemes (e.g., revenue and earnings insurance) provide protection against adverse movements in several specified risks. Their indemnity payoff function resembles that of exotic options with complex contingencies. In this paper, it is shown how option pricing techniques can be used to calculate fair premiums for three existing revenue insurance contracts. The products are sold by private insurance companies, but are reinsured by the US government. It is also shown that the reinsurance contract can be valued by the same technique.

1. INTRODUCTION

Since the seminal work of Black and Scholes (1973) and Merton (1973), the arbitrage principle underlying option pricing theory has been extended to a broad range of real and financial instruments. The extant literature has shown that any security whose payoff is contractually related to the returns on other traded securities can be valued using arbitrage principles. This is the case for standard financial products such as warrants and convertible bonds, as well as contractual agreements such as insurance contracts.

Starting with Merton's (1977) treatment of deposit insurance, numerous authors have applied the arbitrage pricing principle underlying modern option pricing theory to the valuation of insurance products. In particular, this theory has been applied to pricing various forms of life insurance (Brennan and Schwartz 1976; Walden 1985), property/liability contracts (Cummins 1988; Shimko 1992), private mortgage insurance (Kau, Keenan, and Muller 1993), and catastrophe insurance (Cummins, Lewis, and Phillips 1997; Litzenberger, Beaglehole, and Reynolds 1996).[1]

In this paper, we adopt option pricing techniques to calculate the premiums for three existing agricultural revenue insurance contracts with *multiple named perils*. Our model assumes competitive, complete, and frictionless markets. We ignore nonobservability, adverse selection, and moral hazard. The contracts

[1] This body of literature marks the convergence of finance and insurance research as envisioned by Miller (1993) and discussed by Smith (1986) and Garven (1987).

considered are particularly interesting because their underlying risks can be hedged using traded futures and options contracts on yield and price (see Li and Vukina 1998). Turvey (1992) is among the first authors to view a simple crop revenue insurance contract as a standard European put option. More recently, Jung and Ramezani (1999) have shown that the crop revenue coverage (CRC) insurance contract has a path-dependent indemnity with a stochastic deductible. While the present paper has much in common with the foregoing work, it extends it by valuing a broader range of contracts, and the expanded analysis leads to several new insights. The products we consider are based on the same *named perils*, but differ in their deductible and indemnity payoff functions. Each contract offers the policyholder the ability to mitigate specific forms of uncertainty. Using a "plain vanilla" contract as the base case, we show that the added flexibility offered by more complicated contracts can be valued as *embedded* options.

Because agricultural risks are highly correlated, insurance companies cannot mitigate their risk exposure by diversifying across policyholders, and hence they are exposed to catastrophic risks. For this reason, private insurance companies have been historically reluctant to underwrite such policies without some form of government support. Therefore, the Federal Crop Insurance Corporation (FCIC), a wholly owned government entity subsidizes the premiums paid by farmers, pays part of the insurer's claims processing expenses, and reinsures them against potential underwriting losses.

In general, insurance policies expose the insured to the risk of policy nonperformance (a form of default risk). Nonperformance risk influences the demand for insurance (Doherty and Schlesinger 1990), further weakening the market for agricultural insurance. Government grant of reinsurance makes these insurance policies more valuable, as it reduces (or eliminates) the nonperformance risk of the contracts. Thus, the benefit of shifting this risk to the government accrues to the insurers and policyholders.[2]

The government granted reinsurance is an implicit subsidy provided to the insurers. We show that the payoffs to the reinsurance contract can be represented as a portfolio of options on traded assets. Our analysis is the first to provide a direct estimate of the cost of this implicit subsidy based on the risk-neutral valuation principle.

The contents of the paper are organized as follows. In the next section, insurance policies are cast as complex derivatives. In Section 3, we show that the payoffs to the reinsurance contracts are identical to a portfolio of simple options. In Section 4, we introduce the underlying stochastic processes. In Section 5, we value the insurance and reinsurance contracts using Monte Carlo simulation. In the final sections, we discuss the simulation results and draw our conclusions.

[2] The reinsurance contracts we consider are similar to the excess-of-loss catastrophe reinsurance contracts recently proposed by the Clinton administration. However, the proposed catastrophe reinsurance contracts have a single source of risk (e.g., earthquake). Cummins, Lewis, and Phillips (1997) propose a methodology for pricing the latter contracts. For an overview of reinsurance contracts, see Cantor, Cole, and Sandor (1997) and Litzenberger, Beaglehole, and Reynolds (1996).

2. REVENUE INSURANCE CONTRACTS AS COMPLEX DERIVATIVES

We begin with a mathematical representation of the various indemnity payments and show that their structure is similar to path-dependent options on several underlying stochastic processes. The contracts we consider are single-year multirisk insurance contracts. Because of the complexity of the indemnity payment, a closed-form pricing formula cannot be derived in every case. When this occurs, we employ Monte Carlo simulation to numerically calculate the premiums. Our framework is easily extended to other insurance contracts containing multiple sources of risk.

Although our analysis would be the same for all existing crop insurance contracts, we consider only contracts on corn because it has the most significant market value. Fix the start of the production year, t, at 15 March. This is the date the insurance contract is purchased. Let T be 1 December of the same year, when the indemnity is paid.

Multiple peril crop insurance (MPCI) is the most common form of coverage and has been the principal means of managing yield risk since the 1930s. This product provides protection against all risks affecting output. Producers receive indemnity payment only if their yield falls below an established minimum guarantee. Although, such outcomes may be accompanied by upward price swings, shortfalls in yield are valued at a preestablished base price, rather than the potentially higher harvest price. Consequently, the contract protects only against shortfalls in output and not revenue (price times output).

The MPCI's minimum guarantee MG is established when the contract is entered at t as

$$MG = c\bar{Y}p_t, \tag{1}$$

where $50\% \leqslant c \leqslant 75\%$ (in 5% increments) is the fraction of the actual production history $\bar{Y}$ (bushels per acre) that the producer chooses to insure. The base price p_t is a regional price average, which is calculated at date t and remains unchanged throughout the life of the policy.

At the contract settlement date T, the calculated revenue R_{MPCI} is

$$R_{\text{MPCI}}(Y_T) = Y_T p_t, \tag{2}$$

where Y_T is the realized output. If R_{MPCI} is below MG, the contract pays an indemnity I_{MPCI}:

$$I_{\text{MPCI}}(Y_T) = \max\{0,\ MG - R_{\text{MPCI}}(Y_T)\}. \tag{3}$$

The indemnity payment is identical to a European put option with MG acting as the strike price and R_{MPCI} as the underlying asset. To value MPCI, a stochastic process for output Y_T must be specified. Any proposed process would have zero drift, because the *growth rate* of yield over one planting period is zero. Under a lognormal output process, the Black and Scholes (1973) formula would be appropriate provided that a traded asset highly correlated with yield exists (i.e., CBOT futures contracts on yield; see Li and Vukina 1998). MPCI serves as our base case insurance contract. The other contracts we consider improve on MPCI by offering additional flexibility, which we value as embedded options.

Under the current *actuarial method* of insurance rate-making, the premiums for MPCI are set equal to a multiple of the average historical ratio of indemnity payouts to insured value (typically, 27 years of data). This pricing method is based exclusively on the first moment of the yield distribution and ignores all higher-order moments. Moreover, futures prices are not used to set the base price; hence, the method of setting premiums is not forward looking. Josephson, Lord, and Mitchell (2000) provide further details of the actual MPCI rate-making process. A final criticism (Borch 1985) of actuarial methods of rate-making is that these methods ignore the market price of risks. For example, a variety of *ad hoc* discount rates are used (Garven 1987). Our approach addresses this concern.

Income protection (IP) expands on MPCI by guaranteeing revenue, rather than yield alone. The guarantee is the product of the prevailing futures market price at the time of planting (for delivery at harvest) multiplied by the base yield. Indemnity payments are made only if revenues at harvest fall below the minimum guarantee.

The minimum guarantee for IP is identical to that of MPCI (equation (1)), but the base price p_t is calculated as the product of the average $\bar{F}_t$ of February daily settlement futures price (December delivery) times the farmer's chosen price coverage ($\alpha = 95\%$ or 100%): $p_t = \alpha \bar{F}_t$. The calculated revenue R_{IP} is

$$R_{\mathrm{IP}}(p_T, Y_T) = Y_T p_T, \tag{4}$$

where the output price p_T is the product of the average $\bar{F}_T$ of the November daily settlement price (December delivery) times the price coverage: $p_T = \alpha \bar{F}_T$. If the calculated revenue falls below the minimum guarantee, an indemnity is paid:

$$I(p_T, Y_T) = \max\{0,\ MG - R_{\mathrm{IP}}(p_T, Y_T)\}. \tag{5}$$

The indemnity payment is a European put option that depends on two *named perils*. Because p_T is the arithmetic average of futures prices, the indemnity is path dependent and a closed-form valuation formula is not available. Note that under IP the farmer may have a low yield, but still be ineligible for an indemnity payment if the harvest price is sufficiently high. Moreover, with IP, the farmer is exposed to the *correlated risks* of both output and price. We integrate this correlation into our pricing methodology.

The *actuarial method* of insurance rate-making for IP uses linear regression to model the level of price and yield for each locality. Given the model parameters and the current levels of the price and yield, an empirical revenue distribution is constructed by randomly sampling from the residuals of the fitted regression models (10 000 draws). For each run of the simulation, the indemnity is calculated as the shortfall in projected revenue relative to the guaranteed revenue. Premiums are set equal to the average of the simulated indemnities plus an arbitrary 20% loading factor to ‘‘accommodate future price fluctuations’’ (General Accounting Office 1998, p. 71). While this procedure accounts for the historical relation between prices and yield, it ignores other important factors. For example, simulated indemnities, which are paid after the harvest season, are not discounted to the time of planting when premiums are set.

Crop revenue coverage (CRC) takes IP one step further by adding *replacement coverage* to the guarantee, should the price at harvest exceed the price at planting. Consequently, CRC provides upward price protection, where the guaranteed floor is the maximum of two separate guarantees: the minimum guarantee MG and the harvest guarantee HG.

While MG is set on the planting date t (equation (1)), HG is not determined until T:

$$HG(p_T) = c\bar{Y}p_T, \tag{6}$$

where $\bar{Y}$ and p_T are as in IP, but CRC places limits on the output price: $(p_t - L) \leqslant pT \leqslant (p_t + L)$, where $L = \$1.50$ for corn. Note that while CRC improves on the guaranteed floor provided by IP, it forces the policyholder to forfeit price increases beyond L. This tradeoff is explicitly priced in our framework.

The maximum of MG and HG is defined as the final guarantee FG. Jung and Ramezani (1999) have shown that FG is equivalent to $c\bar{Y}$ shares of a European ‘‘collar option’’, written on p_T, with floor p_t and ceiling $p_t + L$:

$$\begin{aligned} FG(p_T) &= \max\{MG,\ HG(p_T)\} \\ &= c\bar{Y}[p_t + C(p_T, p_t) - C(p_T, p_t + L)], \end{aligned} \tag{7}$$

where $C(x, k) = \max\{0, x - k\}$ is the terminal date payoff to a European call option written on an underlying process x with strike price k. The call is a path-dependent Asian option, whose underlying is the arithmetic average of November futures prices for delivery in December.

The calculated revenue R_{CRC} is as in equation (4), but p_T is subject to the price limit L:

$$R_{\mathrm{CRC}}(p_T, Y_T) = Y_T[p_t - L + C(p_T, p_t - L) - C(p_T, p_t + L)]. \tag{8}$$

Hence, R_{CRC} is equivalent to a random number Y_T of shares of a European ‘‘collar option’’, written on p_T, with floor $p_t - L$ and ceiling $p_t + L$. If R_{CRC} falls below FG, the contract makes an indemnity payment:

$$I_{\mathrm{CRC}}(p_T, Y_T) = \max\{0,\ FG(p_T) - R_{\mathrm{CRC}}(p_T, Y_T)\}. \tag{9}$$

The indemnity is identical to that of an option whose terminal payoff is the difference between two European collar options. Note that the strike price is also stochastic.

For these contracts, the averaging of futures prices ‘‘smoothes’’ large daily fluctuations and removes the likelihood of price manipulation. For similar reasons, the historical yield $\bar{Y}$ is averaged over a 4–10 year period. Rothschild and Stiglitz (1976) have shown that in a competitive market a *separating equilibrium* can be achieved by offering a menu of contracts that allows the insured to sort into different risk classes. For the contracts we consider, the parameters c and α determine the effective deductibles, allowing the insured farmers to signal their type. While the deductibles address adverse selection, insurance companies must monitor production activity and assess output quality to minimize moral hazard.

The *actuarial method* of insurance rate-making for CRC assumes that the yield and harvest price are independently normally distributed. Historical data are used to estimate the parameters of these distributions. CRC premiums are the sum of three components. The first is the "yield risk" component, which accounts for the probability that actual yield is lower than the insured yield, conditional on the harvest price being less than the base price. The second is the "upward price risk" component, which accounts for the probability that harvest price is higher than the base price, while actual yield is lower than the insured yield. The third is the "revenue risk" component, which accounts for the probability that actual revenues are below the guaranteed revenue, while both price and yield are below their base level (General Accounting Office 1998).

The CRC premiums are therefore a function of the conditional probabilities obtained from the assumed yield and price distributions. However, because the two distributions are assumed to be independent, the correlation between yields and prices is ignored (General Accounting Office 1998, p. 58). Moreover, unlike the IP procedure, CRC uses the harvest price instead of futures price, which implies that the resulting premiums are not forward looking.

3. REINSURANCE AS A PORTFOLIO OF OPTIONS

Insurance companies that underwrite the above contracts reinsure their exposure through the Federal Crop Insurance Corporation (FCIC). Unlike the traditional "excess of loss" reinsurance, where losses above a prespecified threshold are fully insured (subject to a possible cap), the FCIC reinsurance contract requires the companies to retain responsibility for a *portion* of their risk exposure, thereby minimizing the potential for adverse selection.[3] For the retained portion, underwriting losses and gains are shared between the company and FCIC, based on a complicated payment schedule. The FCIC also reimburses the company for a portion of its administrative and operating expenses.

The reinsurance contract is a single-period contract that expires on date T. For each contract underwritten, the insurance company collects a premium V at t, but the contract exposes the company to the risk of paying an indemnity I at T.

[3] The FCIC allows insurance companies to place their outstanding contracts in three fund categories: (1) Assigned Risk Fund; (2) Development Fund; and (3) Commercial Fund. Each fund offers a different "coverage limit" and "stop-loss reinsurance payment schedule". The coverage limits detail the company's limitations in ceding its risk exposure to the FCIC. All remaining risk after cession is reinsured by the FCIC, and the FCIC agrees to pay losses above certain trigger points.

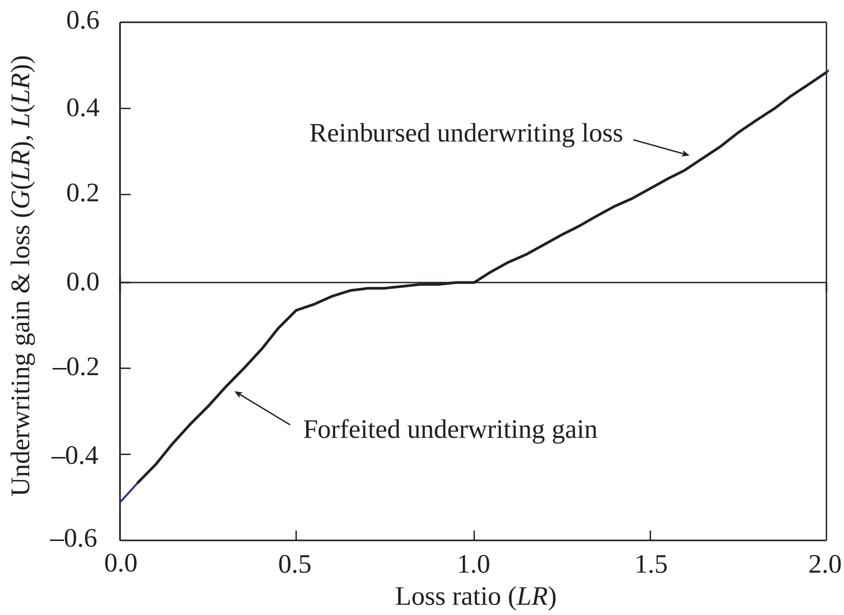

FIGURE 1. Reinsurance payoff for crop revenue coverage (CRC) with a net book premium (NBP) of $1.00.

Define the loss ratio as $LR = I/NBP$, where $NBP = V(1 - AO)$ is the net book premiums, with AO (a percentage) the administrative and operating expense subsidy. Under reinsurance, FCIC claims a portion of the company's underwriting gains and reimburses a portion of its underwriting losses. When $LR < 1$, the portion of the gain that is recovered by FCIC is $G(LR)$. Similarly, when $LR > 1$, the portion of the loss that is reimbursed by FCIC is $L(LR)$.

Figure 1 plots the net gains $G(LR)$ and losses $L(LR)$ per dollar of net book premium NBP for a single contract. As the plot shows, the sharing rule between FCIC and the insurance company is asymmetric and highly nonlinear, where large losses (gains) are subject to higher reimbursement (recovery). The Appendix shows that the sharing rule consists of several layers of excess of loss reinsurance, with each layer marked by a different loss ratio that is equivalent to a standard European spread option.

Because of their payoff complexity, no analytical solution exists, and we proceed numerically. Specifically, when the loss ratio LR is less than 1.0, the net gain is a put spread:

$$G(LR) = NBP[\delta_{0.65}P(1.00) + (\delta_{0.50} - \delta_{0.65})P(0.65) + (\delta_{0.00} - \delta_{0.50})P(0.50)] \geqslant 0, \tag{10}$$

where $P(k)$ is a European put option (i.e., $\max\{0, k - LR\}$). Equation (10) is simply the payoff to a portfolio of put options, where the δ_{LR}'s are the FCIC mandated exposures (or, equivalently, the number of shares) to each put, with $\delta_{0.65} = 6\%$, $\delta_{0.50} = 30\%$, and $\delta_{0.00} = 89\%$. Therefore, the FCIC's claim on the insurance company increases at an accelerating rate as LR decreases. Similarly, when $LR > 1$, the net loss is a call spread:

$$L(LR) = NBP[\delta_{1.00}C(1.00) + (\delta_{1.60} - \delta_{1.00})C(1.60) + (\delta_{2.20} - \delta_{1.60})C(2.20) + (\delta_{5.00} - \delta_{2.20})C(5.00)] \geqslant 0, \quad (11)$$

where $C(k)$ is a European call option and the δ's are the exposures to each call, with $\delta_{1.00} = 43\%$, $\delta_{1.60} = 57\%$, $\delta_{2.20} = 83\%$, and $\delta_{5.00} = 100\%$. All losses above $LR = 5.00$ are fully reimbursed.

To summarize, the insurance company holds a portfolio consisting of single-period insurance and reinsurance contracts. At the settlement date T, the company pays out indemnity I_k and receives (pays) reinsurance payment. Hence, the net payment to this portfolio, denoted by NP_k, is

$$NP_k(I, LR) = L_k(LR) - G_k(LR) - I_k, \quad (12)$$

where the subscript k denotes different contracts: $k =$ MPCI, IP, or CRC. The equation demonstrates how the FCIC-sponsored reinsurance contract reduces a company's risk exposure by capping the maximum loss, as without reinsurance, the net payout is simply $-I_k$. Specifically, as indemnities I_k increase, the firm begins to experience losses as its LR climbs above 1.0, implying $L_k(LR) > 0$ and $G_k(LR) = 0$. Note that the present value of $NP_k(I, LR)$ is the premium that would be charged in an efficient, competitive, frictionless, and complete market.

4. SPECIFICATION OF THE UNDERLYING RISK PROCESSES

As noted earlier, commodity markets offer futures and option contracts that enable the insurers to hedge their exposure to yield and price risks. In principle, futures price risk can be perfectly hedged using existing instruments. This is not the case with insured yield, as the marketed yield instruments (futures and option) are based on aggregate yield at the state or national level, rather than a specific farm's yield. As a consequence, the insurer is exposed to yield basis risk. To assess the impact of basis risk on premiums, we calculate the premiums and reinsurance costs for a representative farm with yield distribution identical to the aggregate yield at the national level.

The contracts under consideration have three underlying state variables, representing the evolution of corn futures price F, aggregate yield Y, and the individual farm's yield Y_i. We assume these state variables follow a trivariate correlated geometric Brownian motion

$$\left.\begin{aligned} F_T &= F_t \exp\big[(\mu_F - \tfrac{1}{2}\sigma_F^2)(T-t) + w\sigma_F\sqrt{T-t}\,\big], \\ Y_T &= Y_t \exp\big[(\mu_Y - \tfrac{1}{2}\sigma_Y^2)(T-t) + z\sigma_Y\sqrt{T-t}\,\big], \\ Y_{i,T} &= Y_{i,t} \exp\big[(\mu_{Y_i} - \tfrac{1}{2}\sigma_{Y_i}^2)(T-t) + v\sigma_{Y_i}\sqrt{T-t}\,\big], \end{aligned}\right\} \quad (13)$$

where μ_F, μ_Y, μ_{Y_i} and σ_F, σ_Y, σ_{Y_i} are the instantaneous drift and volatility (constants) of futures price, aggregate yield, and individual yield, and w, z, and v are correlated Brownian motions with correlation $\rho_{FY}\,\mathrm{d}t = \mathsf{E}[\mathrm{d}w, \mathrm{d}z]$, $\rho_{FY_i}\,\mathrm{d}t = \mathsf{E}[\mathrm{d}w, \mathrm{d}v]$, and $\rho_{YY_i}\,\mathrm{d}t = \mathsf{E}[\mathrm{d}v, \mathrm{d}z]$.[4] We have ignored the possibility of jumps and mean reversion in both the futures price and yield processes.

5. VALUING THE INSURANCE AND REINSURANCE CONTRACTS BY MONTE CARLO SIMULATION

In a risk-neutral environment, the value of the insurance and reinsurance contracts is the discounted value of their expected terminal date cashflow. The expectation is under the risk-neutral measure, and discounting occurs at the risk-free rate r. Monte Carlo simulation approximates the expectation with the arithmetic average of the terminal payoffs taken over a finite number $n = 1, \ldots, N$ of simulated price paths (see Boyle 1977). The calculated value of the insurance premium is

$$V \approx \mathrm{e}^{-r(T-t)}\left(\frac{1}{N}\sum_{n=1}^{N} I(p_T^n, y_T^n)\right), \tag{14}$$

where $I(p_T, Y_T)$ is given by equation (3), (5), or (9). Similarly, the calculated value of reinsurance Re is

$$Re \approx \mathrm{e}^{-r(T-t)}\left(\frac{1}{N}\sum_{n=1}^{N}[L(LR^n) - G(LR^n)]\right), \tag{15}$$

where $G(LR)$ and $L(LR)$ are given by equations (10) and (11).

Following standard practice, we use Cholesky decomposition to transform the correlated processes in (13) into uncorrelated Brownian motions. Following Black (1976) and Marcus and Modest (1984), we assume the risk-neutral futures price process is a martingale with no drift ($\mu_F = 0$).[5] For the representative farm, $\rho_{YY_i} = 1$, and, for the individual farm, $\rho_{YY_i} \neq 1$. For economic reasons and without loss of generality, we set the correlation between futures price and individual farm yield to zero ($\rho_{FY_i} = 0$), but allow the correlation between futures price and the representative farm to vary ($\rho_{FY_i} \neq 0$).

The production setting is taken to be point input/point output, where the crop is planted at t and harvested approximately 10 months later at T. Hence, the instantaneous growth rate of yield, though positive over many years, is zero

[4] There exists an extensive literature assessing the empirical distribution of crop yields. Just and Weninger (1999) provide a survey of this literature and show that the assumption of normality of yields cannot be rejected. Using nonparametric estimation procedures, Goodwin and Ker (1998) show that crop insurance premiums are not sensitive to the choice of yield distributions. This lack of consensus, particularly because yields cannot assume negative values, leads us to our distributional assumption.

[5] For alternative specifications of the futures price process, see Schwartz (1997).

within any crop year ($\mu_{Y_i} = \mu_Y = 0$). Once planting has occurred, the actual output Y_T is the realization from a fixed distribution, ruling out any supply response to short-run changes in market conditions.

For simplification, we also assume that $\sigma_{Y_i} = \sigma_Y$. This is a reasonable assumption for regions with homogeneous agronomic conditions (like the corn-belt), where yields flow from the same exogenous factors. Lastly, we assume both the aggregate and individual farm yields (bushels per acre) are uncorrelated with the overall economy during a given planting cycle. Hence, their market price of risk is zero. Alternatively, Merton (1998) shows that a tracking portfolio, with error that is orthogonal to the overall market, can be constructed from a portfolio consisting of all traded assets. Further, two tracking instruments for corn yield, yield insurance futures and futures options, are traded on the Chicago Board of Trade (CBOT).

For a given set of parameters, we generate 20 000 simulations for futures price and yield at T. These values are then used in equations (14) and (15) to calculate premiums V and reinsurance value Re.

6. RESULTS

The results of our analysis are shown in Tables 1–3. We present our findings for different sets of parameters and across different insurance contracts. The first part highlights the sensitivity of premiums to a change in parameters, while the second compares the various contracts to demonstrate the value of the embedded options.[6]

The contract's premium V and reinsurance value Re are both linearly increasing in yield and price coverage levels c and α. To obtain an upper bound for both V and Re, we set c and α to their maximum allowable values of 0.75 and 1.00. For the 1997 crop year, FCIC data indicates that aggregate corn yield $\bar{Y}$ was 126 bushels per acre and base price p_t was \$2.60 per bushel. For each run of the simulation, we set the initial futures price equal to the base price ($p_t = F_{t,T}$), so that the contract is "at-the-money" with respect to price. The 1997 Treasury bill rate is used for the risk-free rate ($r = 5.47\%$).[7]

Table 1 shows how the calculated premiums V and reinsurance value Re vary with changes in yield volatility σ_Y. Table 2 presents the same results for changes in futures price volatility σ_F. The calculated values are for a "representative" farm with $\rho_{YY_i} = 1$, $\sigma_Y = \sigma_{Y_i} = 0.04$, and $\bar{Y} = 126$. The selected levels of $\bar{Y}$ represent different degrees of *moneyness*, where $Y_t = c\bar{Y} = 94.5$ bushels/acre is at-the-money.

[6] We assume throughout the insurance company writes a single insurance contract and retains full exposure to that contract. Also, the contract is reinsured and allocated to the FCIC-designated Commercial Fund.

[7] The administrative and operating expense subsidy AO is set at 27% for MPCI and IP, and 23.25% for CRC (Federal Crop Insurance Corporation 1997).

TABLE 1. Premium V and reinsurance value Re versus yield volatility σ_Y.

Yield volatility (%) σ_Y	Expected yield					
	$Y_t = 80$		$Y_t = 100$		$Y_t = 126$	
	V	Re	V	Re	V	Re
Multiple peril crop insurance						
.02	36.18	4.68	0.00	0.00	0.00	0.00
.04	36.19	4.88	0.18	0.10	0.00	0.00
.06	36.19	5.25	0.89	0.45	0.00	0.00
.08	36.22	5.61	1.94	0.88	0.00	0.00
.10	36.37	5.95	3.23	1.34	0.00	0.00
.12	36.71	6.28	4.66	1.81	0.02	0.01
.14	37.19	6.60	6.10	2.28	0.09	0.05
.16	37.86	6.95	7.67	2.74	0.26	0.13
.18	38.64	7.29	9.18	3.18	0.53	0.26
.20	39.60	10.04	10.74	3.63	0.95	0.45
Income protection						
.02	41.36	8.15	13.47	4.36	1.81	1.00
.04	41.26	8.08	12.96	4.21	1.73	0.96
.06	41.27	8.02	12.70	4.12	1.48	0.83
.08	41.34	7.98	12.27	4.01	1.35	0.76
.10	41.43	7.94	12.46	4.04	1.31	0.75
.12	41.71	7.97	12.56	4.05	1.33	0.75
.14	42.02	8.03	12.62	4.05	1.41	0.79
.16	42.44	8.09	13.18	4.19	1.56	0.87
.18	42.93	8.19	13.58	4.28	1.72	0.95
.20	43.48	8.33	14.41	4.49	1.99	1.08
Crop revenue coverage						
.02	55.94	7.46	13.47	3.99	1.81	0.96
.04	56.41	7.60	13.16	3.88	1.73	0.92
.06	56.82	7.77	13.58	3.81	1.48	0.80
.08	57.19	7.94	14.13	3.74	1.35	0.73
.10	57.38	8.07	15.38	3.85	1.31	0.72
.12	57.91	8.27	16.54	3.98	1.36	0.74
.14	58.21	8.43	17.57	4.15	1.52	0.81
.16	58.65	8.67	19.13	4.49	1.87	0.96
.18	59.31	8.90	20.34	4.73	2.29	1.15
.20	59.85	9.12	21.97	5.08	2.94	1.42

The entries in each cell are the calculated premiums V and reinsurance value Re. Each row shows the impact of changes in yield volatility σ_Y on V and Re. A total of 20 000 simulations are performed with yield coverage $c = 75\%$, price coverage $\alpha = 100\%$, base price $p_t = \$2.60$, futures price $F_t = \$2.60$, futures price volatility $\sigma_F = 0.25$, correlation between yield and futures price $\rho_{F_y} = -0.5$, historical yield $\bar{Y} = 126$ bushels/acre, and $r = 5.47\%$.

TABLE 2. Estimated premiums V and reinsurance value Re versus futures price volatility σ_F.

Expected yield	Futures price volatility σ_F	$\rho_{FY} = 1.0$ V	$\rho_{FY} = 1.0$ Re	$\rho_{FY} = -0.5$ V	$\rho_{FY} = -0.5$ Re	$\rho_{FY} = 0.0$ V	$\rho_{FY} = 0.0$ Re
Income protection							
$Y_t = 80$	.20	38.63	6.92	38.97	7.28	39.41	7.57
	.30	43.36	8.59	43.98	8.88	44.58	9.15
	.40	49.36	10.17	50.07	10.47	50.49	10.64
	.50	55.89	11.68	56.35	11.88	56.95	12.08
	.60	62.48	13.00	62.97	13.21	63.49	13.40
$Y_t = 100$	.20	7.57	2.67	9.18	3.15	10.52	3.57
	.30	15.49	4.84	16.69	5.20	17.89	5.55
	.40	23.41	6.79	24.65	7.14	25.80	7.46
	.50	31.84	8.72	32.61	8.94	33.85	9.26
	.60	39.97	10.39	40.86	10.65	41.49	10.83
$Y_t = 126$	.20	0.21	0.13	0.53	0.32	0.87	0.51
	.30	2.64	1.39	3.57	1.81	4.29	2.13
	.40	7.63	3.42	8.71	3.80	9.84	4.19
	.50	14.33	5.57	15.51	5.95	16.35	6.21
	.60	21.67	7.53	22.59	7.85	23.50	8.06
Crop revenue coverage							
$Y_t = 80$	.20	52.84	6.40	52.38	6.76	51.86	7.05
	.30	61.19	8.07	60.37	8.46	59.54	8.81
	.40	68.67	9.73	67.95	10.17	66.73	10.46
	.50	75.79	11.24	74.60	11.58	73.50	11.99
	.60	81.72	12.34	80.61	12.73	79.38	13.20
$Y_t = 100$	.20	7.83	2.48	9.36	2.92	10.60	3.29
	.30	15.76	4.46	16.91	4.78	17.98	5.08
	.40	23.60	6.18	24.79	6.50	25.82	6.74
	.50	31.61	7.71	32.36	7.92	33.46	8.17
	.60	38.79	8.92	39.66	9.15	40.22	9.28
$Y_t = 126$	.20	0.21	0.10	0.53	0.31	0.87	0.49
	.30	2.63	1.33	3.56	1.73	4.29	2.02
	.40	7.52	3.17	8.60	3.52	9.75	3.88
	.50	13.69	4.89	14.87	5.24	15.76	5.51
	.60	19.84	6.12	20.77	6.45	21.81	6.69

The entries in each cell are the calculated premiums V and reinsurance value Re. Each column shows the impact of changes in the correlation between futures price and yield on V and Re. A total of 20 000 simulations are performed with yield coverage $c = 75\%$, price coverage $\alpha = 100\%$, yield volatility $\sigma_Y = 0.04$, base price $p_t = \$2.60$, futures price $F_t = \$2.60$, historical yield $\bar{Y} = 126$ bushels/acre, and $r = 5.47\%$.

Table 1 shows the impact of changes in yield volatility σ_Y on premiums V and the reinsurance value Re for different degrees of moneyness. The relation is almost linear (rising with σ_Y) and becomes U-shaped as the contract goes out-of-the-money. This is a consequence of the complex payoff structure of these contracts.[8]

Table 2 shows that V and Re significantly change with the degree of moneyness. Each column shows that V and Re increase with the futures price volatility σ_F. Similarly, each row shows the impact of the yield–price correlation ρ_{FY}. When the contract is in-the-money, V decreases and Re increases as σ_{FY} increases; otherwise they move together. Our results quantify the conjecture put forward in the General Accounting Office (1998, p. 58) report: ignoring σ_{FY} significantly biases V and Re. Again, similar results are obtained at the individual farm level.

In practice, FCIC sets the CRC premium uniformly above the IP premium (General Accounting Office 1998). Our results indicate the gap in premiums should be related to the degree of moneyness. For example, when σ_F is low, and the insured yield is expected to exceed the historical average (the contracts is out-of-the-money), the CRC premium is higher than the IP premium (and the corresponding Re values). The opposite occurs when the contract is in-the-money and σ_F is high.

We compare our calculated premiums with actual premiums for 1997. The base price ($p_t = 2.60$) is the only parameter common to the two sets of data. Because the historical yield $\bar{Y}$, yield coverage c, and price coverage α associated with the actual premiums are unknown, the premium values cannot be directly compared. However, by normalizing the premiums relative to the maximum insured liability, we can provide a rough comparison.

The distributional characteristics of the ratio of premiums to maximum insured liability (by state) are reported in Table 3. Overall, the mean ratio for our analysis is similar to the mean ratio for the actual premiums. This is due primarily to our comparisons with the aggregated state premiums. However, because our pricing approach is fundamentally different from the models used by FCIC, the higher-order moments diverge significantly.

Several factors can explain the differences found in Table 3. First, our model parametrization is significantly different from the one used by the FCIC.[9] Second, these insurance contracts offer a number of additional flexibilities. Farmers are allowed to replant their insured acreage, collect part of their indemnity before harvest guarantee is known, separate their farm into irrigated or nonirrigated practices with different premium rates, and spread their premium payment over a number of months. These provisions provide valuable options that are not priced in our models. Similarly, premiums may reflect agency costs that are absent from our model.

[8] Individual farm results are available from the authors.

[9] Because our analysis uses a static one-period model, we ignore the FCIC mandated multiperiod sinking fund provision in valuing the reinsurance contract (Federal Crop Insurance Corporation 1997).

TABLE 3. Comparison of actual and model-generated premiums.

State	Policy	N	Mean	SD	Min	Max	Kurtosis	Skewness
Option model	CRC	30	0.0977	0.1009	0.0034	0.2433	−1.5524	0.6048
	MPCI	30	0.0872	0.1094	0.0002	0.2538	−2.0562	0.2374
USA	CRC	796	0.0958	0.0504	0.0386	0.5490	12.9992	2.7022
	MPCI	796	0.0790	0.0466	0.0258	0.4094	7.6253	2.2114
Colorado	CRC	17	0.0762	0.0078	0.0652	0.0957	0.9823	0.7123
	MPCI	17	0.0622	0.0095	0.0507	0.0848	0.3881	0.9597
Illinois	CRC	98	0.0807	0.0468	0.0408	0.4075	25.4524	4.3316
	MPCI	98	0.0708	0.0595	0.0292	0.4094	18.1265	4.0108
Iowa	CRC	100	0.0704	0.0190	0.0467	0.1331	1.9562	1.4285
	MPCI	100	0.0538	0.0182	0.0325	0.1186	2.0740	1.4940
Kansas	CRC	96	0.1048	0.0646	0.0448	0.3697	4.4770	1.9520
	MPCI	96	0.0861	0.0499	0.0332	0.2990	2.9392	1.4524
Michigan	CRC	45	0.1294	0.0431	0.0759	0.2521	0.2028	0.9743
	MPCI	45	0.1172	0.0448	0.0636	0.2291	0.0710	0.9562
Minnesota	CRC	66	0.1063	0.0499	0.0571	0.2583	2.2902	1.7460
	MPCI	66	0.0928	0.0575	0.0392	0.2976	3.2532	1.9302
Missouri	CRC	72	0.1426	0.0402	0.0867	0.3044	4.3620	1.5993
	MPCI	72	0.1222	0.0304	0.0709	0.2240	1.8414	1.0206
Nebraska	CRC	90	0.0758	0.0271	0.0386	0.1875	4.4134	1.7668
	MPCI	90	0.0587	0.0207	0.0258	0.1197	0.2888	0.7647
Ohio	CRC	65	0.0823	0.0327	0.0430	0.2102	5.0986	2.0475
	MPCI	65	0.0606	0.0266	0.0287	0.1655	4.6801	1.9997
Oklahoma	CRC	10	0.1563	0.0903	0.0581	0.3372	0.4948	1.2249
	MPCI	10	0.1226	0.0807	0.0474	0.2828	0.6849	1.3933
South Dakota	CRC	45	0.1285	0.0397	0.0639	0.2278	−0.2617	0.5219
	MPCI	45	0.1172	0.0453	0.0473	0.2367	0.3251	0.5623
Texas	CRC	14	0.0870	0.0131	0.0386	0.1094	−1.0924	0.4946
	MPCI	14	0.0683	0.0057	0.0597	0.0808	0.6272	0.9568

The entry in each cell is the statewide average (across N counties) of the ratio of premiums to maximum insured liability for multiple peril crop insurance (MPCI) and crop revenue coverage (CRC). The "option model" values are averages of calculated premiums V from Table 1 divided by the calculated maximum liability.

7. CONCLUSIONS

This paper values CRC, IP, and MPCI revenue insurance products along with their associated reinsurance contracts as portfolios of exotic and standard options. We utilize Monte Carlo simulation to calculate the premiums for these products under a wide range of parameters. Our model accounts for important factors that have been overlooked in the current rate-making practice, particularly the correlation structure of the state variables. Our analysis shows that the value of the reinsurance contract is a significant fraction of the calculated premiums. Although strong assumptions underlie our model (complete, competitive, and frictionless markets), our analysis provides an estimate of premiums in a competitive equilibrium setting. Our analysis is useful because it provides a benchmark by which the efficacy of other pricing schemes can be measured.

APPENDIX

Reinsurance contract as a portfolio of options

This appendix shows that the reinsurance contract can be represented as a portfolio of standard European options. For each contract, the insurance company collects a premium V at t, but the contract exposes the company to the risk of paying an indemnity I at T. Under reinsurance, the "underwriting loss" is defined as the amount by which the indemnity exceeds the "net book premium", $I - NBP > 0$, where $NBP = V(1 - AO)$, with AO (a percentage) the expense subsidy paid by the FCIC. Conversely, "underwriting gain" is $NBP - I > 0$. Define the loss ratio as $LR = I/NBP$. The "net underwriting gain" $G(LR)$ is the portion of the gain that is recovered by the FCIC when the loss ratio is low ($LR < 1$). Similarly, the "net underwriting loss" $L(LR)$ is the portion of the loss that is reimbursed by the FCIC when $LR > 1$.

The payment schedule on the company's net underwriting gain $G(LR)$ is cumulative and increases as the loss ratio LR decreases. Specifically, if $0.65 \leqslant LR < 1.00$, the company pays $\delta_{0.65} = 6\%$ of its underwriting gain $NBP - I$ to the FCIC:[10]

$$\begin{aligned} G_{0.65}(LR) &= \delta_{0.65}(NBP - I) \\ &= \delta_{0.65} NBP(1.00 - LR). \end{aligned} \tag{A.1}$$

In addition to the payment in equation (A.1), the company must pay $\delta_{0.50} = 30\%$ of all underwriting gains earned over the range $0.50 \leqslant LR < 0.65$. This payment is $\delta_{0.50}(0.65NBP - I)$, which when combined with equation (A.1)

[10] We assume that a single revenue insurance contract is reinsured, and it is allocated to the FCIC-designated Commercial Fund because the bulk of 1997 premiums were allocated to this fund (Glauber 1999). Although not shown, reinsurance values Re under the Development Fund generally exceed that of the Commercial Fund.

results in a cumulative payment of

$$G_{0.50}(LR) = \delta_{0.65}(NBP - 0.65V) + \delta_{0.50}(0.65NBP - I)$$
$$= NBP[0.35\delta_{0.65} + \delta_{0.50}(0.65 - LR)]. \tag{A.2}$$

Finally, in addition to the payment in equation (A.2), the company must forfeit $\delta_{0.00}(0.50NBP - I)$ for all underwriting gains earned over the range $0.00 \leqslant LR < 0.50$. The overall cumulative payment is therefore

$$G_{0.00}(LR) = \delta_{0.65}(NBP - 0.65NBP) + \delta_{0.50}(0.65NBP - 0.50NBP)$$
$$+ \delta_{0.00}(0.50NBP - I)$$
$$= NBP[0.35\delta_{0.65} + 0.15\delta_{0.50} + \delta_{0.00}(0.50 - LR)], \tag{A.3}$$

where $\delta_{0.00} = 89\%$. Thus, net underwriting gains consist of several layers of traditional excess of loss reinsurance, with each layer marked by a different loss ratio.

A similar "layered" payment schedule applies to the net underwriting loss $L(LR)$, which is cumulative and increasing in LR. If $1.00 < LR \leqslant 1.60$, the company is reimbursed

$$L_{1.00}(LR) = NBP\delta_{1.00}(LR - 1), \tag{A.4}$$

where $\delta_{1.00} = 43\%$. If $1.60 < LR \leqslant 2.20$, the cumulative reimbursement is

$$L_{1.60}(LR) = NBP[0.60\delta_{1.00} + \delta_{1.60}(LR - 1.60)], \tag{A.5}$$

where $\delta_{1.60} = 57\%$. If $2.20 < LR \leqslant 5.00$, the cumulative reimbursement is

$$L_{2.20}(LR) = NBP[0.60(\delta_{1.00} + \delta_{1.60}) + \delta_{2.20}(LR - 2.20)], \tag{A.6}$$

where $\delta_{2.20} = 83\%$. Finally, if $5.00 < LR$, the cumulative reimbursement is

$$L_{5.00}(LR) = NBP[0.60(\delta_{1.00} + \delta_{1.60}) + 2.80\delta_{2.20} + \delta_{5.00}(LR - 5.00)], \tag{A.7}$$

where $\delta_{5.00} = 100\%$. Thus, all losses above $LR = 5.00$ are fully reimbursed.

Acknowledgements

We thank J. David Cummins, Diego Garcia, Paul Fackler, Hua He, Robert Merton, Mario Miranda, Mark Rubinstein, Brian Wright, and seminar participants at the University of California (Davis and Berkeley), University of Wisconsin-Madison, and the American Risk and Insurance Association (Vancouver 8/99) for their helpful comments and suggestions. We remain responsible for any errors.

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