

**Valuing Risk Management Tools as Complex Derivatives:
An Application to Revenue Insurance¹**

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Valuing Risk Management Tools as Complex Derivatives:

An Application to Revenue Insurance

Revenue insurance schemes provide protection against declines in production, prices, or both. Their indemnity payment structure resembles options with complex contingencies -- exotic options. Crop Revenue Coverage (CRC) is a privately sold, but government subsidized, insurance product that protects farmers against adverse movements in prices and yield. We show that CRC's indemnity payments involve the exchange of a known quantity of European Collar Options for a random quantity of similar options: an Asian option with stochastic strike price. We use Monte Carlo Simulation to value this exchange of options. Our results provide a new framework for measuring mispricing in this important insurance market.

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Introduction

Revenue insurance products are integral tools for risk management, providing protection against declines in output, prices, or both. Typically, the indemnity payment from Revenue insurance is the difference between an insured revenue level, acting as a strike price, and realized revenues when the policy matures.

In its simplest form, the indemnity payment for revenue insurance resembles that of a simple European option, where the insured revenue level (strike) is predetermined and the realized revenue is the product of two stochastic state variables, output and price. Despite this complication, such options can be valued in a straight forward manner using standard option pricing techniques (Wilmott, Dewynne & Howison 1993).

In more complicated forms, the indemnity payment for revenue insurance may resemble the payoff pattern of a complex derivative security. For example, the insured revenue level may be stochastic and realized revenue may be a complicated function of several stochastic state variables, including output and price.

In this paper, we consider revenue insurance of the latter variety. In particular, we value Crop Revenue Coverage (CRC) insurance, where the insured revenue level (strike) is stochastic and realized revenue is based on the average of the output price (Asian feature). CRC is a privately sold, but government subsidized, insurance product that protects farmers against adverse movements in prices and yield.

We show that CRC's indemnity payments involve the exchange of a known quantity of European Collar Options for a random quantity of similar options, making the strike price stochastic. Because traded futures contracts exits for both yield and prices, we use standard option pricing techniques along with Monte Carlo Simulation to value this exchange of options.

Crop Revenue Insurance

Farming is an inherently risky enterprise and yield and price volatility represent the most significant forms of uncertainty in agriculture. Such risks have provided justification for government involvement in agriculture since the depression era, despite the existence of a variety of market based hedging instruments. Managing farm incomes, stabilizing supplies and prices, and food security considerations have provided the rationale for continued government involvement in agriculture. However, because of recent shifts in policy, government's role in *directly* mitigating risk in agriculture has diminished. Instead, government plays an *indirect* role by subsidizing *crop revenue insurance* plans, which are widely sold to farmers across the nation.¹

The Federal Crop Insurance Corporation (FCIC), a wholly owned government corporation administers these revenue insurance programs and provide support by subsidizing the premiums paid by farmers, paying part of sales and claims processing costs, and re-insuring underwriting losses that may be suffered by private insurance companies. The majority of supported insurance plans insure revenues at prevailing prices at the time of planting. However, the CRC insures revenues at the larger of the planting or the harvest price.²

We show that purchasing this complex option essentially provides the same benefits as purchasing a standard revenue insurance plan and maintaining a price hedge in the commodity markets. Given this favorable feature of the CRC, it is not surprising that CRC has claimed a significant portion of the market and is projected to become the dominant form of revenue insurance.³

Much controversy surrounds the provisions of CRC. These concern the distributional effects of administering and monitoring the new policies and the incentive incompatibility problems that arise from government's re-insurance and subsidization of private insurance companies (Greenberg, January 11, 1998). Developing a theoretically consistent model for determining the CRC premiums is of paramount importance for resolving existing controversies and ensuring the financial soundness of the program over time.

As a recent GAO (1998) study suggests, the current procedures for determining CRC

premiums are ad hoc, complex, and too closely tied to other insurance programs. Moreover, “the CRC plan does not base its rate structure upon the interrelationship between crop prices and farm-level-yields - an essential component of actuarially sound rate setting.” That is, current methods ignore the correlation structure of yield and prices. The GAO report concludes that current CRC premiums may not adequately protect the government from financial losses due to re-insurance

The purpose of this paper is to show that the CRC indemnity pay-outs are identical to that of a complex derivative security - a path dependent option whose payoff is a function of at least two stochastic processes. Our model provides a benchmark for measuring mispricing of the CRC insurance. Mispricing has significant consequences in terms of increased costs to the government. Because of the existence of re-insurance through FCIC, systematic under-pricing of CRC may necessitate a large government bail-out of the crop insurance industry. Similarly, over-pricing may exclude many farmers from participation in the program and may lead to increased regulation of the crop insurance industry. In either case, the cost to the government will rise, and contrary to the government's stated objective, its role in agricultural markets will expand.

The CRC as an Exotic Option

Under standard revenue insurance plans, the farmer's revenue guarantee is established by multiplying the prevailing prices at the time of planting by the farmer's historical average yield per acre. Farmers receive indemnity payments only if their actual revenue at harvest falls below the guaranteed revenue. If a price increase is offset by a fall in production, or vice versa, no payment is made. CRC differs from these plans in that the revenue guarantee is recalculated at harvest time, and the indemnity payment is based on the higher of the prices at harvest or planting time.

In this section we provide a mathematical representation of the CRC's indemnity payment and show that CRC's complicated payment structure is a path dependent option whose payoff is a function of at least two underlying stochastic processes. We use Monte Carlo simulation

to numerically value the CRC premium, because we know of no closed form pricing formula. To keep the paper focused, we limit our numerical analysis to corn, though the proposed framework is applicable to all crops covered under CRC.

Let t denote both the CRC purchase date and the time that planting decisions are made. Let T be the harvest time, when the indemnity payment is made. At t farmers select their *Yield Coverage Level*, $c \in [.50, \dots, .75]$, in 5% increments, and their *Price Percentage Coverage*, $\alpha = 0.95$ or 1.00 . These parameters allow the farmer to select her desired level of exposure to yield and price risk.

Denote the farm's Actual Production History (APH) by Y^* , and the sequence of past daily (settlement) futures prices for a contract with delivery date T by $F(t - n, T), \dots, F(t, T)$.⁴ Note that both these variables are known to the farmer and the insurance company at t .

Fix the start of corn production year, t , at March 15, and let T be December 1 of the same crop year. At t the Minimum Guarantee (MG) is calculated as $MG(p_b) = c Y^* p_b$, where c is the yield coverage level, Y^* is the Actual Production History, and p_b is the “base price,” defined as the product of the price percentage (α) and the average of February Chicago Board of Trade (CBOT) corn futures prices for delivery at T .⁵

Similarly, at T , the Harvest Guarantee, HG , is calculated as $HG(p_h) = c Y^* p_h$, where c and Y^* are as above and p_h is the Harvest Price, which is defined as the average of November CBOT corn futures prices for delivery at T multiplied by price percentage (α).

The Final Guarantee, FG , is determined by combining the expressions for MG and HG and accounting for the program's price limit, L , placed on p_h .⁶

$$FG(p_h) = \max[MG(p_b), HG(p_h)] = c Y^* \max[p_b, p_h]$$

where $p_h \in [p_b \pm L]$. We explicitly incorporate the price limits on p_h , into the definition of FG using “cap” and “floor” options (Hull 1997). Let $C(x, k) = \max(0, x - k)$ denote the terminal payoff to a European call option on an underlying process x with strike price k , then FG is:

$$\begin{aligned}
FG(p_h) &= c Y^* \max[p_b, p_h] \\
&= c Y^* [p_b + \max[0, p_h - p_b] - \max[0, p_h - (p_b + L)]] \quad (1) \\
&= c Y^* [p_b + C(p_h, p_b) - C(p_h, p_b + L)]
\end{aligned}$$

The calls in equation (1) are path dependent (i.e., Asian) options, whose underlying is the arithmetic average of November futures prices for delivery date T .⁷ The term in square brackets in equation (1) is a European “Collar Option,” written on p_h , with floor p_b and ceiling $(p_b + L)$. Furthermore, the product of coverage level, c , and the farm's average historical yield, Y^* , determines the number of Collar Options that the insurance policy grants to the farmer.

Denote the farm's actual yield at T by Y_T .⁸ The farmer's Calculated Revenue is defined as $R = Y_T p_h$, where the price is again limited to $p_h \in [p_b \pm L]$. Incorporating the price limit into the definition of Calculated Revenues leads to:

$$R(p_h, Y_T) = Y_T [p_b + C(p_h, p_b - L) - C(p_h, p_b + L)] \quad (2)$$

where again the term in square bracket is a European Collar option, written on p_h , with floor $(p_b - L)$ and ceiling $(p_b + L)$. Because Y_T is not observed until harvest, the number of Collar options in (2) is a random variable at the time the policy is written.

If the farm's Calculated Revenue (2) falls below the Final Guarantee (1), CRC makes up the difference by paying an Indemnity Payment, I , to the farmer. Otherwise no payment is made. The Indemnity Payment is:

$$I(p_h, Y_T) = \max[0, FG(p_h) - R(p_h, Y_T)] \quad (3)$$

The formulation in (3) shows that the CRC indemnity payment is identical to that of an option whose terminal payoff is the difference between two European Collar Options. Figure (1)

depicts the terminal payoff to these collars when $c = .75$, $Y^* = 126$ bushels/acre, $p_b = \$2.60$ per bushel, and corn's price limit is $L = \$1.50$ per bushel. Given these parameters, the realized value of Y_T (set equal to 100 in the figure) determines the position of the revenue curve and the level of indemnity payment.

One method for valuing the complex CRC option in (3) is to solve, using a set of boundary conditions, the multivariate Partial Differential Equation (PDE) that describes the evolution of I as a function of p_h , Y_T , and $T - t$. Such PDE is difficult to solve analytically, because of its complicated boundary conditions, the path dependent nature of p_h , and the existence of cross-partial terms involving harvest price, farmer's yield, and time.⁹

Absent an analytical solution, the PDE describing the value of CRC insurance may be solved using a variety of computationally intensive numerical schemes (Wilmott, et. al. 1993). One such scheme is Monte Carlo simulation, a method that is widely used in academia and industry to solve similar problems (Boyle 1977).¹⁰

Corn Futures Price and Yield Processes

The current procedure for setting CRC premiums ignores the impact that yields may have on crop futures prices (GAO). Addressing this concern, we model the correlation structure between yield and futures prices using a joint log-normal distribution. The empirical literature suggests that the log-normal distribution is a reasonable model for both yields and futures prices (Tirupattur, Hauser & Chaherli 1996).

We calculate the CRC premiums for two types of insured units. The first is a representative individual farm with yield distribution identical to the aggregate yield at the national level. The second is an individual farm, where the yield distribution may be different and only partially correlated with aggregate yield. In this case, three state variables - the futures price, aggregate yield, and the individual farm's yield - determine the value of CRC contract. We assume the state variables follow a tri-variate log-normal process (Tirupattur, et. al. 1996). Specifically, the futures price ($F(\cdot)$) and aggregate yield (Y) follow the correlated geometric Brownian motions:

$$\begin{aligned} dF/F &= \mu_F dt + \sigma_F dW \\ dY/Y &= \mu_Y dt + \sigma_Y dZ \end{aligned} \quad (4)$$

where μ_F , μ_Y and σ_F , σ_Y are the drift and volatility terms of corn futures prices and aggregate yield, and dW , dZ are correlated Brownian motions with correlation coefficient $\sigma_{FY} dt = E[dW, dZ]$.¹¹ We further assume that the individual farm's yield, Y_i , is correlated with aggregate yield and follows a geometric Brownian motion:

$$dY_i/Y_i = \mu_{Y_i} dt + \sigma_{Y_i} dX \quad (5)$$

where μ_{Y_i} and σ_{Y_i} are the drift and volatility terms of the individual farm's yield and dX is correlated with dZ : $\sigma_{Y_i} dt = E[dZ, dX]$.

Valuing CRC by Monte Carlo Simulation

In a risk neutral environment, the value of the CRC indemnity payment is the discounted value of its expected terminal date cash flow. The expectation is under the risk neutral measure, and discounting occurs at the non-stochastic risk free rate r (Hull 1977, Wilmott, et. al. 1993). Then the value of the indemnity payment, V , is

$$V = e^{-rT} E^* [I(p_h, Y_T)] \quad (6)$$

where E^* is the expectation operator under the risk-neutral measure.

Monte Carlo simulation approximates the expectation with a simple arithmetic average of the indemnity payments taken over a finite number of simulated price paths $n = 1, 2, \dots, N$ (Boyle 1977). The calculated value of the CRC indemnity premium is

$$V \approx e^{-rT} \left[\frac{1}{N} \sum_{n=1}^N I(p_h^n, Y_T^n) \right] \quad (7)$$

where p_h^n and Y_T^n , $n = 1, \dots, N$, are obtained from simulations that use the risk neutral probability measure as the underlying distribution.

For large values of N , equation (7) provides a good approximation of the true value of the premium. Indeed, the rate of convergence is σ/\sqrt{N} , where σ is the standard deviation of the terminal date Indemnity payment. In general, the rate of convergence is slow. For example to reduce the rate by a factor of 2, N would have to increase by a factor of 4, implying high accuracy will require lengthy computation time. Alternatively, reducing σ by half would achieve the same result without the need for additional sampling. Several techniques exist to reduce σ , such as antithetic, control variate, stratified sampling, or importance sampling. We implement the antithetic technique, the simplest of the four (Boyle 1977, Bratley, Fox & Schrage 1987).¹²

Simulation of the State Variables

For the purpose of simulating the terminal value of the state variables, the *risk neutral* diffusion processes for yield (aggregate and individual) and futures prices can be restated in terms of uncorrelated Brownian motions using Cholesky decomposition:¹³

$$\begin{aligned} dF/F &= \mu_F dt + \sigma_F dW \\ dY/Y &= \mu_Y dt + \sigma_Y (\rho_{FY} dW_1 + \sqrt{1-\rho_{FY}^2} dW_2) \end{aligned} \quad (8)$$

where dW_1 and dW_2 are uncorrelated Brownian motions. Using Cholesky decomposition again, we can write the individual farm's yield process as:

$$dY_i/Y_i = \mu_{Y_i} dt + \sigma_{Y_i} (\rho_{Y_i Y} \rho_{FY} dW_1 + \rho_{Y_i Y} \sqrt{1-\rho_{FY}^2} dW_2 + \sqrt{1-\rho_{Y_i Y}^2} dW_3) \quad (9)$$

where dW_i , $i=1, 2, 3$ are uncorrelated Brownian motions. For the representative national farm $\rho_{Y_i Y} = 1$ and for individual farm $\rho_{Y_i Y} \neq 1$. When $\mu_{Y_i} = \mu_Y$, $\sigma_{Y_i} = \sigma_Y$, and $\rho_{Y_i Y} = 1$, equations (8) and (9) are identical and there are only two state variables. In setting the CRC premiums, the stochastic process assumed by FCIC sets $\rho_{FY} = 0$ and $\rho_{Y_i Y} = 1$. The consequence of these assumptions is quantified when we present our results.

Following Black (1976), Marcus & Modest (1984), and Marcus & Modest (1986), we model the futures price as a martingale with zero drift ($\mu_F = 0$).¹⁴ Additionally, because of the 30 day averaging, only the last month of the yield and futures price need to be simulated. Letting $T_0 = T - 30$ be the 30th day before expiration of the futures contract and $\Delta t_k = 1$ for $k = 1, \dots, 29$, the futures sample path follows recursively:

$$F_{T_0+k} = F_{T_0+k-1} \exp[-0.5 \sigma_F^2 \Delta t_k + \sigma_F \sqrt{\Delta t_k} \epsilon_{1,k}] \quad (10)$$

where

$$F_{T_0} = F_t \exp[-0.5 \sigma_F^2 (T_0 - t) + \sigma_F \sqrt{T_0 - t} \epsilon_{1,0}] \quad (11)$$

and $\epsilon_{0,k}$, $k = 0, \dots, 29$, are distributed $N(0, 1)$.

Following Marcus & Modest (1984), the production setting is taken to be point input-point output, where the crop is planted at t and harvested at T without interim production decisions. Therefore, the growth rate of yield, though positive over many years, is zero for any given crop year ($\mu_{Y_i} = \mu_Y = 0$). Once planting has occurred, actual yield is the realization from a fixed distribution, ruling out any supply response to short run changes in

market conditions.

For simplification we also assume that $\sigma_{Y_i} = \sigma_Y$. This is a reasonable assumption for regions with homogeneous agronomic conditions (like the corn belt), where yields flow from the same exogenous factors such as weather. Under these assumptions, the date $T_0 + k$ yields are:

$$\begin{aligned} Y_{T_0+k} &= Y_{T_0+k-1} \exp[-0.5 \sigma_Y^2 \Delta t_k + \sigma_Y \sqrt{\Delta t_k} (\rho_{FY} \epsilon_{1,k} + \sqrt{1-\rho_{FY}^2} \epsilon_{2,k})] \\ Y_{T_0} &= Y_t \exp[-0.5 \sigma_Y^2 (T_0-t) + \sigma_Y \sqrt{T_0-t} (\rho_{FY} \epsilon_{1,0} + \sqrt{1-\rho_{FY}^2} \epsilon_{2,0})] \end{aligned} \quad (12)$$

Similarly, for the individual farm's yield,

$$\begin{aligned} Y_{i,T_0+k} &= Y_{i,T_0+k-1} \exp[-0.5 \sigma_Y^2 \Delta t_k \\ &\quad + \sigma_Y \sqrt{\Delta t_k} (\rho_{Y_iY} \rho_{FY} \epsilon_{1,k} + \rho_{Y_iY} \sqrt{1-\rho_{FY}^2} \epsilon_{2,k} + \sqrt{1-\rho_{Y_iY}^2} \epsilon_{3,k})] \\ Y_{i,T_0} &= Y_{i,t} \exp[-0.5 \sigma_Y^2 (T_0-t) \\ &\quad + \sigma_Y \sqrt{T_0-t} (\rho_{Y_iY} \rho_{FY} \epsilon_{1,0} + \rho_{Y_iY} \sqrt{1-\rho_{FY}^2} \epsilon_{2,0} + \sqrt{1-\rho_{Y_iY}^2} \epsilon_{3,0})] \end{aligned} \quad (13)$$

where $\epsilon_{j,k}$'s, $j = 1, 2, 3$, $k = 0, \dots, 29$, are iid $N(0, 1)$. For a given set of parameters, equations (10 - 13) are used to generate $n = 1, \dots, 20,000$ simulations for futures price and yield at T . These values are then used in equation (7) to calculate the CRC premium.

Results

The choice of model parameters is a critical decision for the calculation of CRC premium in

(7). For this reason, we use a range of values for the key parameters, specifically the volatility and correlation of the state variables. The selected range of values include estimates we obtained from actual data and those reported in the literature (Marcus & Modest 1984, Tirupattur, et. al. 1996).

The CRC premium is linearly increasing in yield and price coverage percentage. To obtain an upper bound on the premiums, we set these parameters at their maximum allowable values: $c = 0.75$ and $\alpha = 1.00$. For the 1997 crop year, U.S. Department of Agriculture (USDA) data indicates that the aggregate corn yield was 126 bushels per acre ($Y^* = 126$), and we set the base price at \$2.60 per bushel ($p_b = \2.60). For each run of the simulation, we set the initial futures price equal to the base price ($p_b = F(t, T)$), but allow the initial yield to differ from the Actual Production History ($Y(t) \neq Y^*$). The 1997 Treasury Bill rate, as reported by the Federal Reserve Bank, is used for the risk free rate ($r = 5.47\%$). Our estimates of the CRC premium are based on 20,000 Monte Carlo simulations and various combination of parameters.

Table (1) presents the premiums for an “average farm:” $\rho_{YY} = 1$, $\sigma_Y = 0.04$ and $Y^* = 126$.¹⁵ Note that since yield evolves according to a geometric Brownian motion with zero drift, $Y(t)$ is the expected yield at harvest, $E[Y(T)] = Y(t)$. As Table (1) shows, for a given ρ_{FY} and $Y(t)$, premiums rise with the volatility of futures price, σ_F . The table also shows that premiums are significantly higher when expected yield falls short of the average historical yield or vice versa. This follows by noting that the indemnity payoff is $\max(0, FG - R)$, where FG and R are proportional to the average historical (Y^*) and realized ($Y(T)$) yields, respectively. When $Y(T)$ drops below Y^* (deep in-the-money), $FG - R$ increases, implying higher premiums. The opposite occurs when $Y(T)$ climbs above Y^* (deep out-of-the-money). Moreover, a rise in the volatility of futures price, σ_F , leads to relatively larger increases in premiums, when expected yield exceeds the average historical yield. Figure (2) provides a three dimensional plot of the CRC premium for the “average farm.”¹⁶

Perhaps the most interesting result reported in Table (1) relates to the influence of the correlation of yield and futures price on premiums. Our results show that ρ_{FY} has a significant impact on the estimated premiums. Figure (3) provides a graph of premium versus ρ_{FY} for the hypothetical average farm. It is clearly seen that the CRC premium is a decreasing function of ρ_{FY} . The figure also indicates that thirty day averaging of the futures price diminishes the effect

of correlation on premiums.

Table (2) shows a very interesting relation between premiums and the yield volatility (σ_Y) for the hypothetical average farm. Columns one, two and three represent a farm with expected output below, equal to and exceeding the farm's historical average. In all three cases, the relation between premium and yield volatility is "U" shaped. From Figure (2) and noting that the initial futures price was set equal to the base price of \$2.60 per bushel, the three cases illustrate the impact of volatility on the premium when the option is "out of the money" (from the perspective of the policy holder). Although not included in table (2), the relationship becomes linear and increasing when the option is "in the money" (yield levels less than 100 bushels per acre).

Table (3) presents estimates of CRC premiums for an individual farm with output correlation of yields, $\rho_{YIY} = 0.5$. Patterns similar to those of Table (1) are repeated: the premium rises with σ_F , falls with ρ_{FY} , and increases when expected yield ($Y_i(t)$) is less than the farm's average historical yield (Y^*). Comparison across Tables (1) and (3) shows that the premium falls as ρ_{YIY} rises. Figure (4) depicts the relationship between the premium and ρ_{YIY} . Again, it is apparent that thirty days averaging of the futures price reduces the influence of ρ_{YIY} on the estimated premiums.

Table (4) provides data on the actual CRC premiums for corn for 1997 crop year.¹⁷ These data show that premiums vary widely across each state and the country as a whole. Average premiums are closely lumped in states in the corn belt and generally rise with distance away from the corn belt. Unfortunately the level of yield and price coverage associated with these data are unknown, though they will surely fall below our assumed values of $c = 75\%$ and $\alpha = 100\%$. It would be interesting to back out the parameters of our model from a more complete set of data. This is an area for further research.¹⁸ Nevertheless the existing data can be compared to our estimates, since ours represent an upper bound for the CRC premiums.

Comparison of the values in Table (4) with our estimates in Table (1) show that, for reasonable levels of price and yield risk, actual premiums far exceed our estimated premiums. Actual premiums approach our estimates only under conditions of grave uncertainty: high

futures price volatility ($\sigma_f > 30\%$), zero yield-price correlation ($\rho_{fY} = 0$), and low expected yield relative to the farm's historical average.

Several factors can explain the gap between our estimates and the actual premiums in Table (4). First, our volatility and correlation parameters may significantly underestimate the true value of these parameters. Though an upward adjustment of these parameters would narrow this gap, the empirical literature supports our choice of parameter values. In fact, the GAO report (1998) (page 35) shows that farm's insured under CRC have a lower average σ_{y_i} than non-insured farms.

In similar vein, actual premiums may reflect a “jump” component, accounting for the probability of “large” shocks to the futures price or the yield processes (a catastrophe). Catastrophe premiums, however, should be zero since government re-insures the CRC insurance providers for the excess of claims payment over the collected premiums. Moreover, the price limit L also protects the insurance companies from “large” price movements. In any case, the majority of farmers hold separate, government subsidized, catastrophe insurance. The CRC insurance policy offers farmers a number of additional flexibilities - options. In practice, farmers are allowed to re-plant (or plant late) their insured acreage, collect part of their indemnity before harvest guarantee is known, separate their farm into irrigated or non-irrigated practices with different premium rates, and spread their premium payment over a number of months. These provisions provide valuable options that are not priced by our model.

The FCIC reimburses the insurance companies for a large fraction of their administrative costs (GAO). It is possible that part of the remaining administrative costs are passed along in the form of higher premiums, which may also explain the estimated gap in premiums. Along this same lines, actual premiums may contain a component that protects the providers for moral hazard problems that arise from insuring yield, which may require significant amount of care from the farmer.¹⁹

Finally, in a perfectly functioning and complete financial market with no transaction costs, taxes or other frictions, farmers can replicate the CRC coverage by hedging in the derivatives and commodity markets. This is essentially the setting assumed in our model. Though the real

world is far from this abstraction, our model still provides a theoretically consistent benchmark to measure the costs associated with such market imperfections.

Conclusions

This paper considers the pricing of crop revenue coverage insurance as an exotic option. We use a preference-free valuation approach and Monte Carlo simulation to value CRC insurance under a wide range of parameter values. Our model is very general, accounting for important factors that have been overlooked in the extant literature, particularly the correlation structure of yields and prices.

Data shows that actual CRC premiums far exceed those suggested by our model. We identify a number of explanations for the divergence between actual premiums and our model values. Further investigation of these factors is likely to be an exciting area for future research.

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Table 1: Estimated Premiums for a Representative Farm (\$/Acre)

Expected Yield	Future Volatility σ_F	Correlation $\rho_{FY} = -1.0$	Correlation $\rho_{FY} = -0.5$	Correlation $\rho_{FY} = 0$
$Y(t) = 100$.20	7.87	9.31	10.60
	.25	11.78	13.16	14.35
	.30	15.73	17.13	18.32
	.35	19.84	20.82	22.13
	.40	23.74	25.16	25.77
	.45	27.61	28.62	29.66
	.50	31.44	32.34	32.87
	.55	35.24	36.07	36.80
	.60	38.57	39.63	40.43
$Y(t) = 126$.20	0.21	0.48	0.85
	.25	1.10	1.68	2.21
	.30	2.71	3.56	4.28
	.35	4.88	5.83	6.98
	.40	7.68	8.66	9.78
	.45	10.30	11.59	12.66
	.50	13.73	14.93	15.87
	.55	16.61	18.05	18.64
	.60	19.66	20.79	21.63
$Y(t) = 140$.20	0.02	0.08	0.17
	.25	0.25	0.45	0.75
	.30	0.91	1.39	1.87
	.35	2.16	2.79	3.46
	.40	3.79	4.85	5.42
	.45	6.17	6.86	7.94
	.50	8.50	9.32	10.56
	.55	10.84	12.07	12.76
	.60	13.48	14.27	15.57

Simulation results ($n = 20,000$) with $\rho_{YY} = 1$, $c = 75\%$, $\alpha = 100\%$, $\sigma_Y = 4\%$,
 $F(t) = p_b = \$2.60$, $Y^* = 126$ bushels/acre, and $r = 5.47\%$.

Table 2: Indemnity Premium versus σ_y for a Representative Farm (\$/Acre)

Yield Volatility (%) σ_Y	Expected Yield $Y(t) = 100$	Expected Yield $Y(t) = 126$	Expected Yield $Y(t) = 140$
2	13.41	1.82	0.54
4	13.02	1.67	0.50
6	13.69	1.48	0.42
8	14.28	1.43	0.37
10	15.36	1.32	0.33
12	16.38	1.39	0.34
14	17.63	1.52	0.33
16	19.07	1.87	0.44
18	20.36	2.38	0.53
20	21.93	3.00	0.84

Simulation results ($n = 20,000$) with $\rho_{YY} = 1$, $c = 75\%$, $\alpha = 100\%$, $\sigma_F = 25\%$, $\rho_{FY} = -0.5$,
 $F(t) = p_b = \$2.60$, $Y^* = 126$ bushels/acre, and $r = 5.47\%$.

Table 3: Estimated Premiums for an Individual Farm (\$/Acre)

Expected Yield	Future Volatility σ_F	Correlation $\rho_{FY} = -1.0$	Correlation $\rho_{FY} = -0.5$	Correlation $\rho_{FY} = 0$
$Y_i(t) = 100$.20	9.31	9.98	10.59
	.25	13.11	13.77	14.39
	.30	17.13	17.65	18.19
	.35	20.91	21.20	22.08
	.40	25.08	25.68	25.77
	.45	28.60	29.67	29.64
	.50	32.12	32.77	33.37
	.55	36.31	36.46	36.68
	.60	39.24	39.91	40.20
$Y_i(t) = 126$.20	0.50	0.68	0.88
	.25	1.61	2.00	2.29
	.30	3.53	3.99	4.31
	.35	5.83	6.39	6.84
	.40	8.66	9.23	9.76
	.45	11.79	12.21	12.71
	.50	14.84	15.30	16.05
	.55	17.73	18.39	18.82
	.60	20.81	21.44	21.79
$Y_i(t) = 140$.20	0.07	0.11	0.17
	.25	0.47	0.57	0.71
	.30	1.33	1.52	1.85
	.35	2.84	3.13	3.47
	.40	4.68	4.98	5.60
	.45	6.95	7.48	7.86
	.50	9.47	9.96	10.27
	.55	11.97	12.41	13.03
	.60	14.27	15.05	15.37

Simulation results ($n = 20,000$) with $\rho_{YY} = 0.5$, $c = 75\%$, $\alpha = 100\%$, $\sigma_Y = 4\%$,
 $F(t) = p_b = \$2.60$, $Y(t) = Y^* = 126$ bushels/acre, and $r = 5.47\%$.

Table 4: Statistics on the Actual CRC Premiums for Corn (1997)

State	Premium/Acre (V)	σ_v (%)	Min (V)	Max (V)	Total Acres
Colorado	16.82	2.36	13.35	20.58	221,757
Illinois	15.92	3.79	11.37	39.80	727,091
Indiana	15.96	3.82	9.89	30.52	395,170
Iowa	14.72	2.44	10.70	21.48	3,056,544
Kansas	14.78	2.99	9.76	27.17	428,973
Michigan	22.10	5.81	14.26	35.39	91,411
Minnesota	19.01	5.42	13.12	35.85	769,781
Missouri	22.64	4.85	15.66	42.57	193,347
Nebraska	14.24	3.54	9.54	30.42	2,505,218
Ohio	15.34	2.98	10.62	23.14	213,678
Oklahoma	18.66	4.58	9.67	25.20	8,929
South	16.62	3.68	10.80	26.73	595,611
Texas	21.57	2.68	17.26	27.15	42,542
U.S.A	17.57	2.95	14.24	22.64	9,250,052

Source: Data obtained from USDA

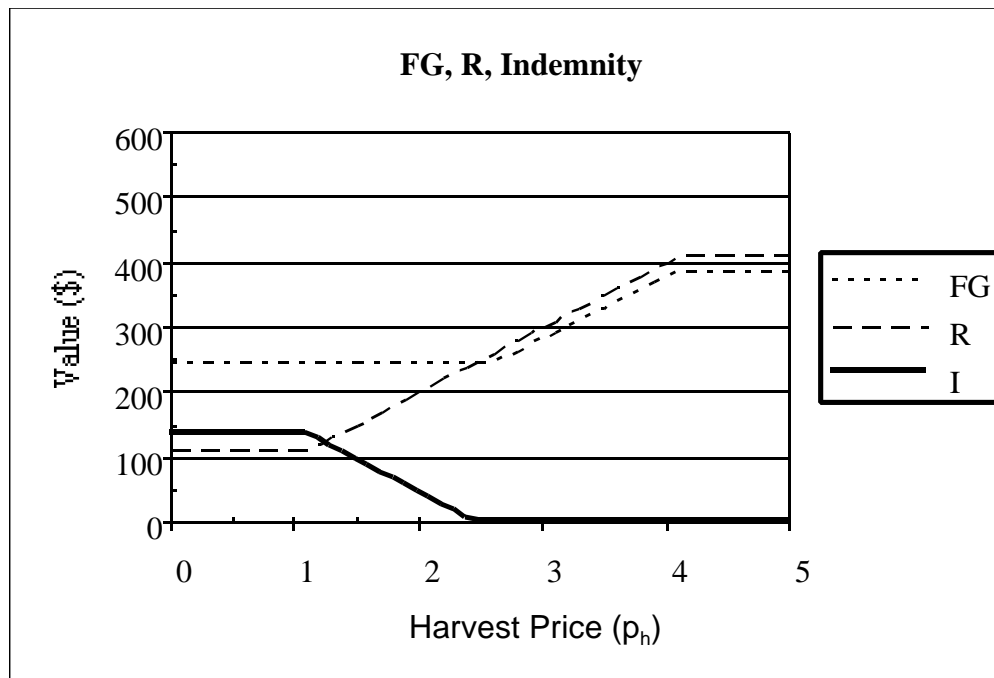


Figure 1: Terminal Date CRC Indemnity Payoff
FG = Fixed Guarantee, *R* = Revenue, *I* = Indemnity

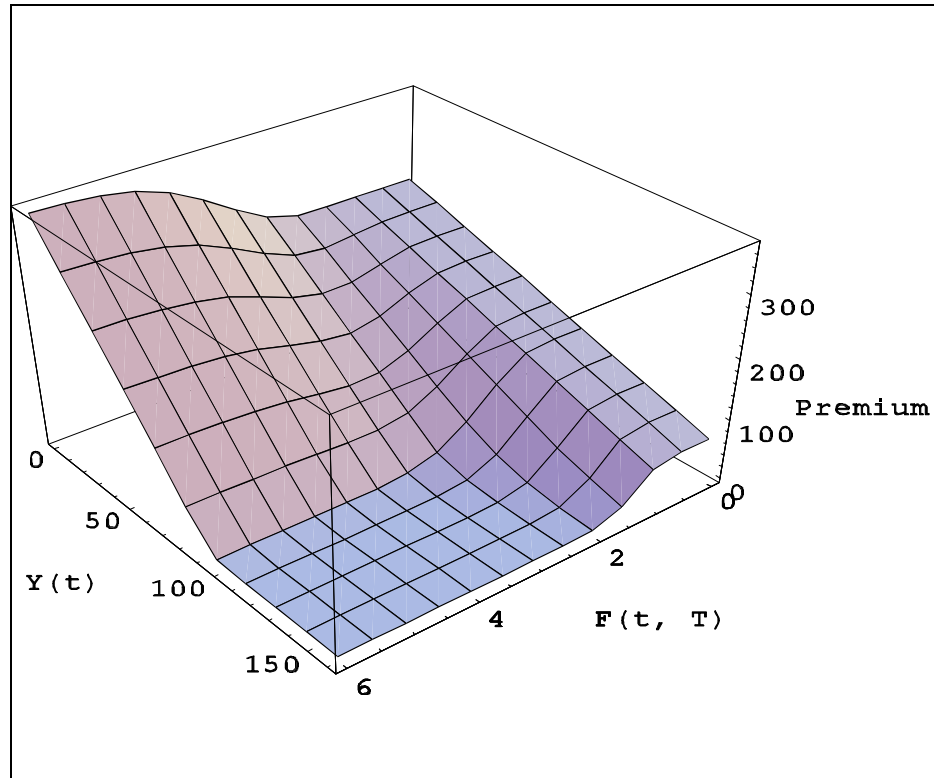


Figure 2: CRC Premium as a Function of Yield and Futures Price

$Y(t)$ = yield at t , and $F(t, T)$ = futures price at t for maturity at T

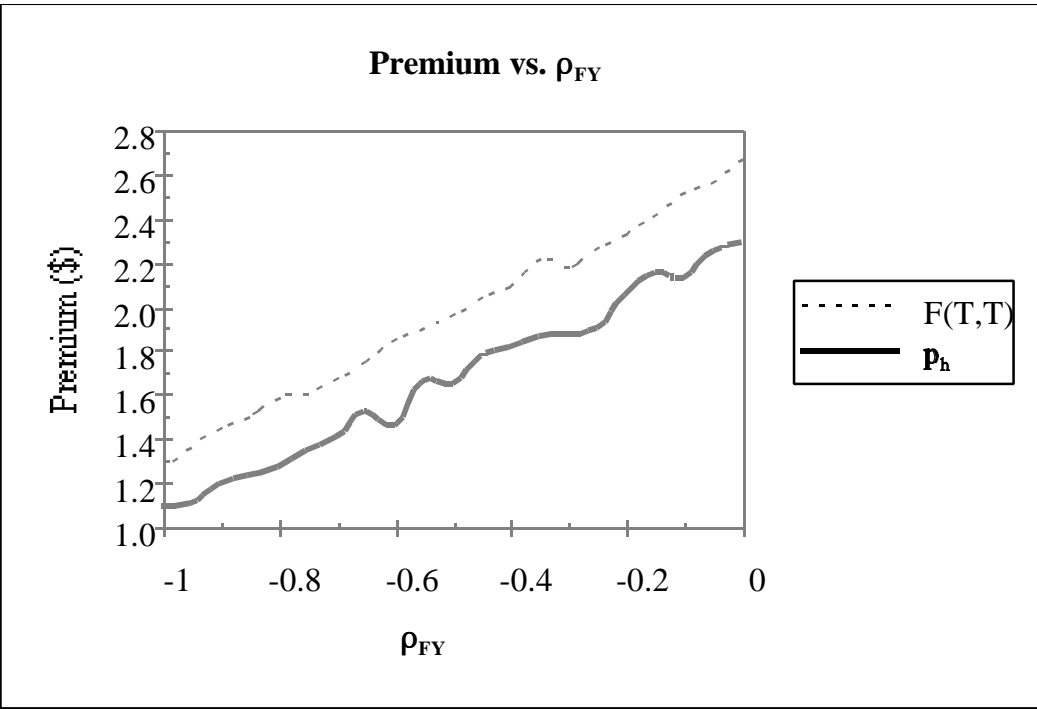


Figure 3: CRC Premium as a Function of Yield and Futures Price
Harvest price = p_h , Terminal Futures price = $F(T,T)$

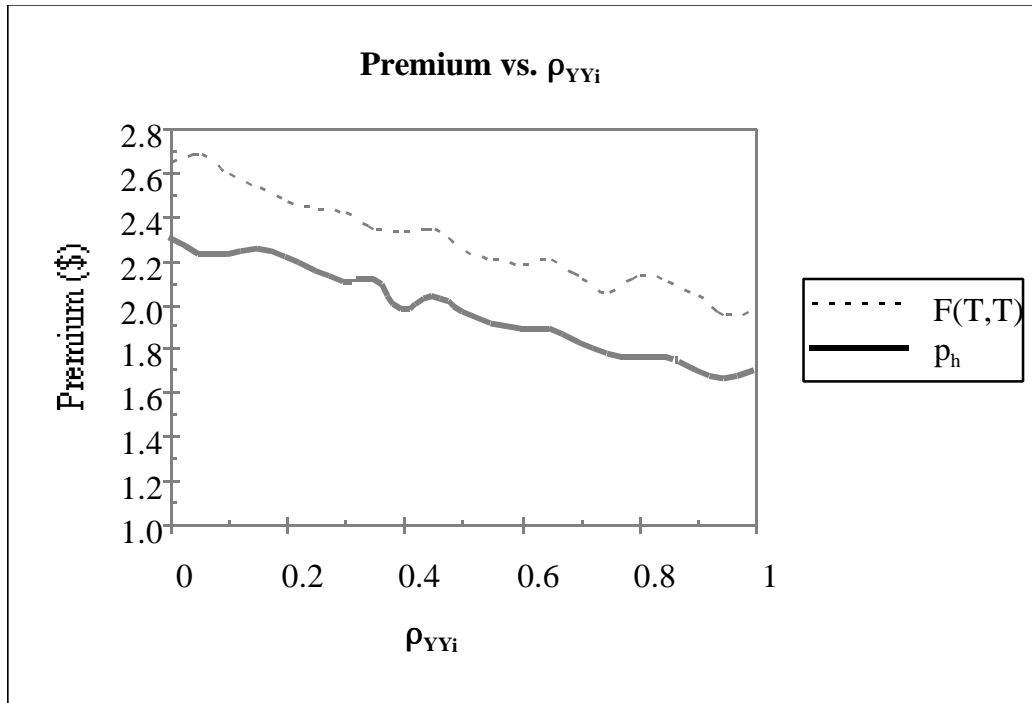


Figure 4: CRC Premium as a Function of Yield Correlation

Harvest price = p_h , Terminal Futures price = $F(T,T)$.

1. A detailed description and comparison of these programs can be found in the recent General Accounting Office report (GAO, (1998)).
2. According to the GAO (1998) study the CRC's costs to the government will likely be significantly more than other plans, because CRC has both higher reimbursements rates for administrative expenses, and it exposes the government to higher underwriting losses, specially when widespread crop losses are coupled with rising prices.
3. Currently the program covers corn, cotton, sorghum, soybeans, and wheat in 18 states. Plans for extension to other crops and geographic areas have been approved by FCIC. The GAO study indicates that CRC was 99.95% of the total acreage covered by the three programs in 1997. CRC also had a 36% loss ratio (claims payments / total premiums), which was the highest among the three plans.
4. APH is the farm's average historical yield, which is often taken from production records that have been maintained for various farm support programs. County level yield data may be substituted in cases where farm specific APH is unknown.
5. In practice farmers may be able to purchase CRC insurance before t . For simplicity we assume that they chooses to postpone such decision until t . Rational behavior dictates an optimal use of this free delay option. Hence from the farmer's perspective p_b is determined at t .
6. Note that the price limit, L , reduces the exposure of both the insurance company and the farmer to large price movement at harvest time. Similarly, thirty day averaging of prices reduce exposure to daily price volatility.
7. Asian options are described in detail in Hull (1997) and Wilmott et. al. (1993).
8. Y_T is referred to as the Production to Count. A farm's output may be appraised or adjusted for quality.
9. We note it is possible to obtain a solution by setting the cross-partial terms equal to zero. Such restrictions significantly reduce the usefulness of the proposed framework.
10. It is widely recognized that Monte Carlo simulation is the most efficient procedure to use in cases with multiple state variables and a path dependent payoff function (Hull (1997) and Wilmott, et. al. (1993)).
11. Modeling the futures price process by a geometric Brownian motion was proposed by Black (1976). Using the same model, Marcus & Modest (1984) study the farmer's production decisions and Marcus & Modest (1986) evaluate price support programs. Schwartz (1998) presents other models for the futures price process and describes their merits. For convenience, we assume that σ_F is time invariant. Duffie (1989) discusses the merits of this assumption.

12. Antithetic variance reduction exploits the symmetry inherent in the $N(0, 1)$ distribution. For each draw ϵ in $N(0, 1)$, both it and its mirror image $-\epsilon$ are used in generating 2 distinct sample paths, one corresponding to ϵ and the other corresponding to $-\epsilon$. Thus, N simulations, effectively yields $2 \times N$ data. Additionally, because each pair $(\epsilon, -\epsilon)$ are perfectly negatively correlated, the variance of the simulated terminal date indemnity may be reduced. For detail see (Boyle 1977 and Bratley, et. al. 1987).

13. Cholesky decomposition is used to create correlated variables using independent variates (see Abramowitz & Stegun (1972) and Tong (1990)). Let x_1 and x_2 be independent standard normal variables. Define

$$\begin{aligned} y_1 &= x_1 \\ y_2 &= \rho x_1 + \sqrt{1-\rho^2} x_2 \end{aligned}$$

Then y_1 and y_2 have a bivariate normal distribution with correlation ρ and $y_i \sim N(0, 1)$, $i = 1, 2$. Let

$$\frac{dS_i}{S_i} = \mu_i dt + \sigma_i dW_i$$

$i = 1, 2$, be two Brownian motions with correlation $\rho dt = E(dW_1, dW_2)$. Using Cholesky decomposition we can write the two processes in term of uncorrelated Brownian motions. Let

$$\frac{dS}{S} = \left[\frac{dS_1}{S_1}, \frac{dS_2}{S_2} \right]$$

$\mu = [\mu_1, \mu_2]$, $dZ = [dZ_1, dZ_2]$, and Σ be a 2×2 matrix with $\Sigma_{ii} = \sigma_i^2$ and $\Sigma_{ij} = \rho\sigma_i\sigma_j$, $i, j = 1, 2$. Using the lower triangular matrix K , where $KK' = \Sigma$, S is transformed to

$$\frac{dS}{S} = \mu dt + K dW$$

where dS/S is distributed $N(\mu, \Sigma)$.

14. Because futures contracts are settled daily (marked to market), the value of any contract and its expected dollar returns are zero, see Black (1976) (page 173).

15. The value of Y^* is the actual average of yields for all U.S. farms in 1997, as reported by USDA. We obtain the estimate of $\sigma_Y = 0.04$ from Tirupattur, et. al. (1996) (page 11).

16. Figure (2) indicates that for values of $F(t, T) < 2.0$, the two dimensional plot of premium (V) versus yield (Y) is the familiar "hockey stick" shape of a put option. Similarly, for a fixed level of yield, the plot of premium versus futures price

$(F(t,T))$ is a Collar with its floor and ceiling reversing as yield increases. The values in Table (1) were calculated at the kink in Figure (2).

17. We thank Dr. Joseph Glauber of the USDA for providing us with these data. The premiums collected in 1997 were approximately \$134 million.

18. We thank the referee for pointing out this area of potential research.

19. Our model does not directly address moral hazard problems. Clearly, the indemnity payment to the farmer is maximized when output is zero. However, to receive the maximum payment, the farmer must pay the insurance premium, bear the costs associated with planting the crop, and forego the revenues from the sale of his output. These costs are likely to exceed the indemnity payment. Also, our model shows that in a dynamic world, the farmer has incentives to maintain her yield close to the farm's historical average, because such effort would reduce the volatility of yield and lead to a reduction in CRC premiums.